

# Topology optimization of elastic adhesive joints using the level set approach

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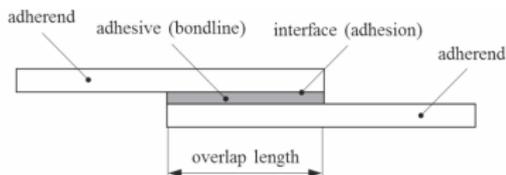
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# Adhesively bonded joints



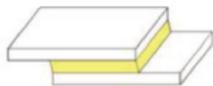
Single lap joint.



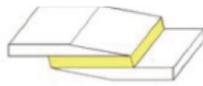
Adhesively bonded aircraft.

- main joint types: mechanical fasteners such as bolts/rivets, welding, clamping, adhesively bonded joints
- adhesively bonded joints are using the polymeric adhesives
- adhesive is a substrate that is capable of strongly and permanently holding two surfaces together
- lightweight aerospace or automotive structures generates the interest in use of adhesively bonded joints of composite, plastic or metal structures

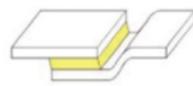
# Basic types of joints



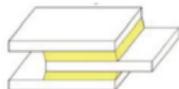
(a) Simple single lap joint



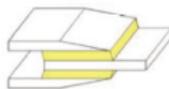
(b) Taper single lap joint



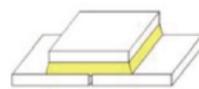
(c) Joggle single lap joint



(d) Simple double lap joint



(e) Taper double lap joint



(f) Strapped single joint



(g) Strapped double joint



(h) Strapped double taper joint

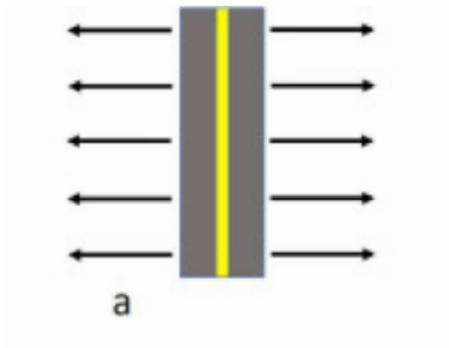


(i) Simple stepped lap joint

Types of joints.

- advantages
  - large surfaces, dissimilar materials and thin substrates
  - reduction the number of parts required and to obtain smooth surface
  - the adhesive has a natural tendency to resist crack propagation and hence provides a significant resistance for fatigue growth from one component to another
  - uniform stress distribution along the bonded area except at edges
- disadvantages
  - limitation imposed on the thickness of the elements that can be joined
  - adhesively bonded joints, once fabricated, cannot be disassembled without damaging the adhesive and/or adherend
  - sensitivity to environment conditions
  - strength is very sensitive to peel and through the thickness stresses

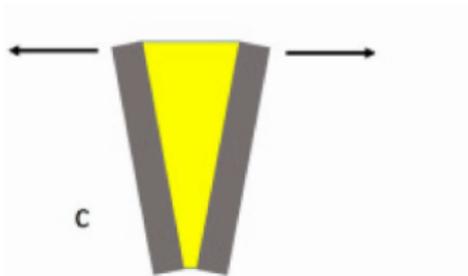
# Adhesive joint stresses



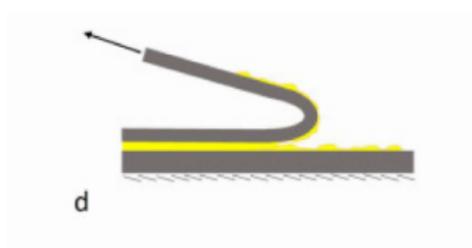
Tensile stress.



Shear stress.



Cleavage stress.



Peel stress.

- bonded joint may fail in many ways
- improve the strength and durability of adhesive joints
- optimisation
  - in different ways and with different objectives
  - reduce or redistribute the mechanical stresses in the adhesive layer in favor of the load capacity
  - chemical (adhesive)
  - geometrical
    - spew fillet, chamfer, modification of adherend geometry: rounding or tapering the adherend edges
  - structural

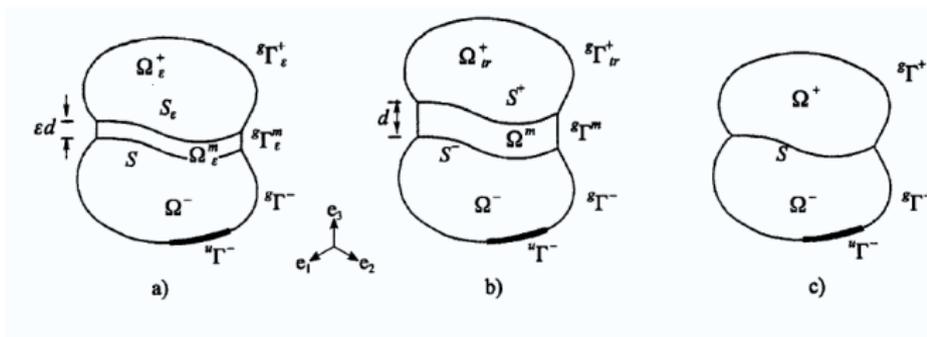
## Volkerson models

- adhesive is modeled as linear-elastic material and deforms only in shearing,
- adherends are elastic and deforms in tension,
- bending effect caused by eccentric load path for single lap joint is neglected,
- peel stress is neglected,
- stresses are constant across thickness.

## Goland-Reissner models

- transverse normal and shear strain in the adherends are negligibly small,
- stress is constant across adhesive thickness,
- adhesive layer is infinitely thin,

# Interface model



The

reference configuration a) physical problem b) the rescaled problem  
c) the limit problem

- $\Omega^+$  and  $\Omega^-$  disjoint open connected domains with piecewise  $C^2$  boundaries  $\partial\Omega^+$  and  $\partial\Omega^-$
- $S = \partial\Omega^+ \cap \partial\Omega^-$  the common part of the external surfaces of the bodies in the plane  $x_3 = 0$
- $\Omega = \Omega^+ \cup \Omega^- \cup S$ .

- adhesive: move  $\Omega^+$  in the  $e_3$  direction of an amount equal to the thickness of the glue
- this thickness  $\varepsilon d$  is assumed small;  $\varepsilon > 0$  denotes the smallness dimensionless parameter and  $d$  is a global characteristic length
- $\Omega_\varepsilon^+ = \{x \in \Omega^+ : x + \varepsilon d e_3\}$ ,
- $\Omega_\varepsilon^m = \{x \in S : x + \varepsilon z d, 0 < z < d\}$ , and  $S_\varepsilon = \{x \in S : x + \varepsilon d e_3\}$ .
- $\Omega_\varepsilon = \Omega_\varepsilon^+ \cup \Omega_\varepsilon^m \cup \Omega_\varepsilon^- \cup S \cup S_\varepsilon$
- $\Omega_\varepsilon^+$  and  $\Omega_\varepsilon^-$  are filled with adherents
- $\Omega_\varepsilon^m$  is filled with adhesive

# Interface model 3

- $V_\varepsilon = \{V \in H^1(\Omega_\varepsilon) : v = 0 \text{ on } \Gamma_\varepsilon^U\}$  - the space of kinematically admissible displacements
- $\epsilon_{ij}(v)$  the linearized strain tensor,
- $A^+$ ,  $A_\varepsilon^-$  and  $A^-$  the fourth-order elastic tensors satisfy the usual smoothness, symmetry and positivity conditions
- body and surface forces:  $f_i \in L^2(\Omega_\varepsilon)$  and  $g_i \in L^2(\Gamma_\varepsilon^g)$

$$a_\varepsilon^+(w, v) = \int_{\Omega_\varepsilon^+} A_{ijkl}^+ \epsilon_{kl}(w) \epsilon_{ij}(v) d\Omega,$$
$$a_\varepsilon^-(w, v) = \int_{\Omega_\varepsilon^-} A_{ijkl}^- \epsilon_{kl}(w) \epsilon_{ij}(v) d\Omega \quad (1)$$

$$a_\varepsilon^m(w, v) = \int_0^{\varepsilon^d} \left[ \int_S A_{ijkl}^m \epsilon_{kl}(w) \epsilon_{ij}(v) dS \right] dx_3, \quad (2)$$

$$a_\varepsilon(w, v) \stackrel{\text{def}}{=} a_\varepsilon^+(w, v) + a_\varepsilon^-(w, v) + \varepsilon a_\varepsilon^m(w, v) \quad (3)$$

$$l_\varepsilon(v) = \int_{\Omega_\varepsilon} f_i v_i d\Omega + \int_{\Gamma_\varepsilon^{g^+}} g_i v_i d\Gamma \quad (4)$$

The displacement  $u_\varepsilon : \Omega_\varepsilon \rightarrow R^3$  of the adhesive joint structure is governed by the variational problem:

Find  $u_\varepsilon \in V_\varepsilon$  satisfying

$$a_\varepsilon(u_\varepsilon, v) = l_\varepsilon(v) \quad \forall v \in V_\varepsilon \quad (5)$$

## Interface model 5

- model (5): domain depends on  $\varepsilon$
- change of coordinates: rescaling the adhesive domain  $\Omega_\varepsilon^m$  by a factor  $\frac{1}{\varepsilon}$  in direction  $e_3$  as well as translating the adherend domain  $\Omega_\varepsilon^+$  by an amount  $(1 - \varepsilon)d$  in the direction of  $e_3$
- in new coordinate system  $y$  the adhesive fills the domain  $\Omega^m = \{y + ze_3 : y \in S \text{ and } 0 < z < d\}$  and the adherents fill the domains  $\Omega^-$  and  $\Omega_{tr}^+ = \{\Omega^+ + de_3\}$ .
- reference configuration of the rescaled bodies is  $S^+ = \partial\Omega_{tr}^+ \cap \partial\Omega^m$ ,  $S^- = \partial\Omega^- \cap \partial\Omega^m = S$  and  $\Omega_{tr} = \Omega_{tr}^+ \cup \Omega^m \cup \Omega^- \cup \Omega^- \cup S^+ \cup S^-$ .
- displacement  $u(\varepsilon, y) = u_\varepsilon(x)$
- solution  $u_0(y)$  to this problem represents the first approximation of the exact solution  $u(\varepsilon, y)$

# Interface model 6: limit problem

$$V_0 = \{v \in L^2(\Omega_{tr}) : v^+ \in H^1(\Omega_{tr}^+), v^- \in H^1(\Omega_{tr}^-), v_{,3}^m \in L^2(\Omega^m), \\ v = 0 \text{ on } \Gamma_{tr}^u, v^+ = v^m \text{ on } S^+, v^- = v^m \text{ on } S^-\} \quad (6)$$

where  $v^+$ ,  $v^m$ ,  $v^-$  are the restrictions of  $v$  to  $\Omega_{tr}^+$ ,  $\Omega^m$ ,  $\Omega^-$ , respectively. The space  $V \subset V_0$ .

$$a^+(w, v) = \int_{\Omega_{tr}^+} A_{ijkl}^+ \epsilon_{kl}(w) \epsilon_{ij}(v) d\Omega, \\ a^-(w, v) = \int_{\Omega^-} A_{ijkl}^+ \epsilon_{kl}(w) \epsilon_{ij}(v) d\Omega, \quad (7)$$

$$a_{33}^m(w, v) = \int_S \left[ \int_0^d A_{i3j3}^m w_{i,3} v_{j,3} dy_3 \right] dS, \quad (8)$$

$$a(w, v) \stackrel{\text{def}}{=} a^+(w, v) + a^-(w, v) + a_{33}^m(w, v) \quad (9)$$

$$l(v) = \int_{\Omega_{tr}^+ \cup \Omega^-} f_i v_i d\Omega + \int_{\Gamma_{tr}^{g^+} \cup \Gamma^{g^-}} g_i v_i d\Gamma \quad (10)$$

The limit problem<sup>1</sup> for (5) is defined as follows:

Find  $u_0 \in V_0$  satisfying

$$a(u_0, v) = l(v) \quad \forall v \in V_0 \quad (11)$$

- for  $\varepsilon \rightarrow 0$   $u(\varepsilon)$  converges to  $u_0$

<sup>1</sup> M. Serpilli et al. International Journal of Solids and Structures 291(2024)

# Topology optimization problem 1

## design variable

$$\Omega \subset D$$

- occupied domain filled with strong and weak material
- $\rho \approx 0$  void  $\rho \approx 1$  solid [phase concentration]
- $u = u(\Omega, \rho)$  is a solution to (11) depending on  $\Omega$

## Cost functional

$$J(u(\Omega, \rho)) = \int_{\Omega} \psi(u) dx + \int_{\Gamma} \tilde{\psi}(u) ds \quad (12)$$

In applications:

- $\psi = \frac{1}{2} \sigma : \epsilon, \tilde{\psi} = 0;$
- $\psi = (\sigma_x/f_x)^2 + (\sigma_y/f_y)^2 - (\sigma_x\sigma_y/f_x f_y)^2 + (\tau_{xy}/f_{xy})^2, \tilde{\psi} = 0;$

# Topology optimization problem 3

Topology optimization problem OP1 (phase field)

Find  $\rho^* \in U_{ad}^\rho$  such that

$$J_\epsilon(\rho^*, u^*) = \min_{\rho \in U_{ad}^\rho} J(\rho, u), \quad (13)$$

- $U_{ad}^\rho = \{\rho \in H^1(\Omega; [0, 1]) : Vol(\Omega) \leq Vol^{giv}\}$

Topology optimization problem OP2 (sharp interface)

find  $E^* \in U_{ad}^E$  such that

$$J_1(E^*, u^*) = \min_{E \in U_{ad}^E} [J_1(E, u) + \gamma Per(E; \Omega)], \quad (14)$$

- $U_{ad}^E = \{E \in BV : Vol(\Omega) \leq Vol^{giv}\}$

# Derivative of cost functional 1

the adjoint state  $p \in V_0$ :

$$a(p, \varphi) = -\left(\int_{\Omega} \frac{d\psi(u)}{du} \varphi dx + \int_{\Gamma} \frac{d\tilde{\psi}(u)}{du} \varphi ds\right) \quad \forall \varphi \in V_0 \quad (15)$$

The shape derivative of the cost functional (14) in direction of the velocity field  $\Theta$  is given by

$$J'_{\Omega} = a'_{\Omega}(u, p) + \tilde{J}'_{\Omega}(u, p) \quad (16)$$

$$a'_{\Omega}(u, p) = \int_{\partial\Omega_r^+} (A_{ijkl}^+ \epsilon_{kl}(u) \epsilon_{ij}(p)) \Theta \cdot \nu ds + \int_{\partial\Omega^-} (A_{ijkl}^+ \epsilon_{kl}(u) \epsilon_{ij}(p)) \Theta \cdot \nu ds + \int_{\partial\tilde{\Omega}} (A_{i3j3}^m u_{i,3} p_{j,3}) \Theta \cdot \nu ds \quad (17)$$

- $H_{mc}$  mean curvature of the boundary
- $\nu$  unit outward normal vector
- $\tilde{\Omega} = S \times [0, d]$

$$\begin{aligned} \tilde{J}'_{\Omega}(u, p) = & \int_{\Gamma} (\psi(u) - fp) \Theta \cdot \nu dx + \int_{\Gamma} (H_{mc} \tilde{\psi}(u) + \\ & \partial_{\nu} \tilde{\psi}(u)) \Theta \cdot \nu ds - \int_{\Gamma} (H_{mc} gp + \partial_{\nu}(gp)) \Theta \cdot \nu ds \end{aligned} \quad (18)$$

Necessary optimality condition:

$$\tilde{J}'_{\Omega}(\Omega, u)(\Omega - \Omega^*) \geq 0 \quad \forall \Omega \in U_{ad}^E$$

The level set function  $\Psi(\cdot) : R^{d+1} \rightarrow R$ :

$$\begin{cases} \Psi(x) = 0 & \text{if } x \in \Gamma \cap D, \\ \Psi(x) < 0 & \text{if } x \in \Omega, \\ \Psi(x) > 0 & \text{if } x \notin \Omega. \end{cases} \quad (19)$$

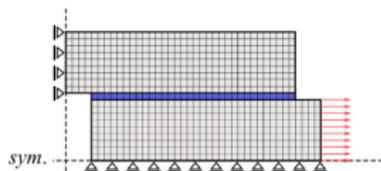
$$\frac{\partial \Psi}{\partial t} + \Theta |\nabla \Psi| = 0, \quad \Psi(0, x) = \Psi_0(x) \text{ in } D, \quad (20)$$

$\Psi_0(x) = \Psi(x, 0)$  is given

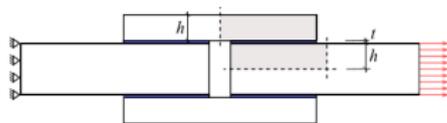
- $\Theta$  is extended on the whole computational domain  $D$
- explicit second order upwind scheme is used
- CFL condition satisfied
- periodic reinitialization

$$l = 60\text{mm}, \quad h = 20\text{mm}, \quad d = 2\text{mm}$$

$$E_1 = 10\text{GPa}, \quad E_2 = 3\text{GPa}, \quad \nu_1 = 0.4 \nu_2 = 0.35$$

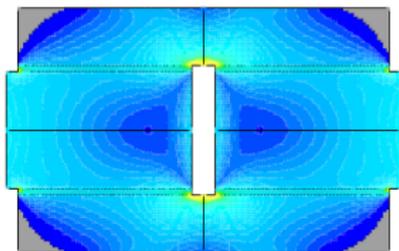


Finite element model.

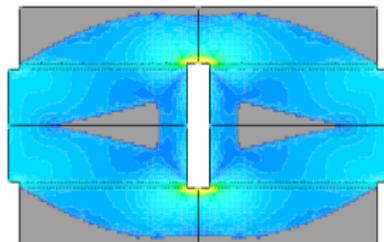


Design domain.

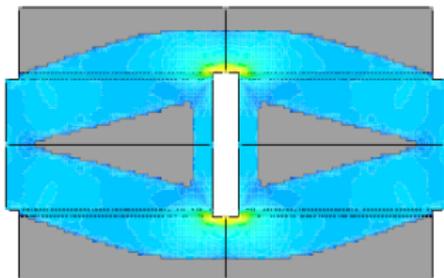
# Example 1: DLJ



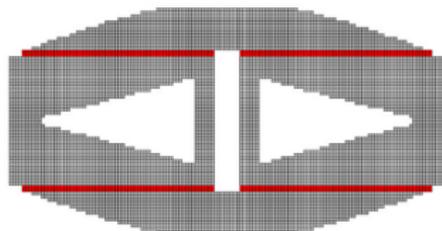
Domain evolution.  $i=10$ .



Domain evolution.  $i=30$ .

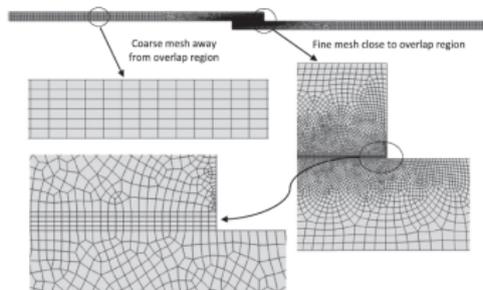


Domain evolution.  $i=55$ .



Optimal topology domain.

# Numerical example 2. SLJ optimization.



SLJ: FE mesh.

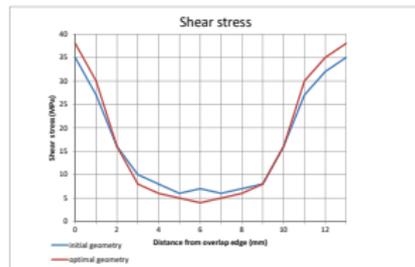
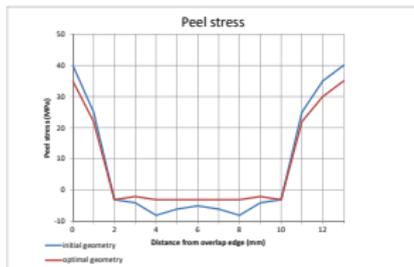


SLJ: Initial topology.



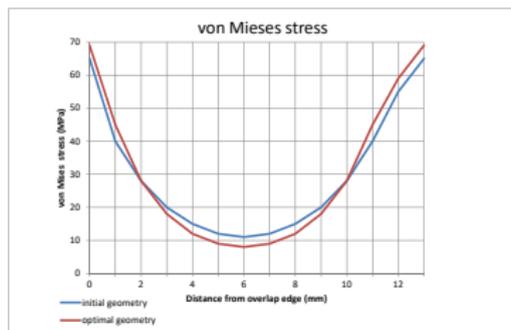
SLJ: Optimal topology.

# SLP - stresses



SLP: peel stress.

SLP: shear stress



# Conclusions

- 1 structural optimization approach has been used to increase mechanical efficiency of adhesively bonded joints
- 2 the optimal topologies may be different especially in terms of manufacturing techniques
- 3 joint model: the adhesive layer should be able to grow and shrink automatically by changing the material of elements at the adhesive-adherend interface. This would allow the adhesive layer to find its optimal geometry
- 4 the load capacity of the optimized joint geometry must be investigated experimentally to evaluate the change in efficiency

**Thank you**

**Q?**

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