Lecture 2 (45 min + 10 min Q&A)

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The Dynamic Turnpike Phenomenon in Conservative Systems: Initial Limit Arcs and Spectral Dichotomy

Abstract:

The Turnpike property is a fundamental feature of optimal control problems over long time horizons, asserting that the optimal trajectory spends the majority of its time near a steady state. While this phenomenon is well-understood for dissipative systems (e.g., heat equations), the case of abstract conservative systems governed by skew-adjoint operators presents unique challenges due to the complete absence of natural energy dissipation mechanisms. This lecture investigates the dynamic turnpike phenomenon in such systems, addressing how the optimal control strategy enforces exponential energy decay in the absence of intrinsic dissipation.

We move beyond static steady-state analysis to resolve the fine structure of the initial transient layer. By employing modal decomposition, we rigorously characterize the Initial Limit Arc—the unique infinite-horizon optimal trajectory evolving on the stable manifold of the optimality system. We establish the uniform exponential convergence of finite-horizon solutions to this limit arc in the energy norm, proving that the transient dynamics are universal and independent of the final time T.

A central contribution of this work is the identification of a Spectral Dichotomy in the decay rates. Through detailed spectral analysis, we reveal two distinct physical regimes based on the interplay between the control regularization parameter and the operator spectrum:
(1)A control-dominated oscillatory regime, where the decay rate is limited by the control cost;
(2)A saturation-limited monotonic regime, where the decay rate is capped by the intrinsic eigenfrequencies.

Finally, we present numerical results for the scalar wave equation and the coupled Timoshenko beam system. These simulations utilize structure-preserving symplectic time integration to rigorously distinguish the control-induced decay from numerical artifacts, thereby confirming the theoretical predictions.

Key topics:

- Optimal control of infinite-dimensional conservative systems
- Transient dynamics and characterization of the Initial Limit Arc
- Spectral dichotomy and analysis of decay rates
- Uniform exponential convergence in the energy norm
- Structure-preserving symplectic integration