Lecture 3 (45 min + 10 min Q&A) Dr Yixin Tan

Level Set Method for Shape and Topology Optimization in Elasticity

Abstract:
A level-set framework is considered for stiffness-oriented design in linear elasticity, where geometry evolution combines boundary variations governed by shape gradients with topology changes driven by topological derivatives. Starting from a traction-driven elasticity model, a compliance-type objective under a volume constraint is formulated, and the first-order optimality structure is derived via the speed method and an adjoint approach. This yields a boundary characterization in terms of an energy-density condition, motivating an equivalent boundary-penalty functional and a regularized formulation based on surface averaging. Topological sensitivity for nucleating small inclusions is then introduced using domain decomposition and Steklov–Poincaré boundary operators, leading to an explicit topological derivative expressed through the polarization tensor and the state/adjoint fields. Descent-type update mechanisms for the level-set function are outlined, including shape-gradient-driven transport, projected topological descent, and a combined strategy enabling simultaneous shape improvement and topology modification within a unified implicit representation.

Key topics:

- Shape derivatives and adjoint-based sensitivity analysis in linear elasticity
- Energy-density optimality condition and boundary reformulation
- Topological derivatives via Steklov–Poincaré operators
- Level-set evolution driven by shape gradients and topological descent
- Combined shape–topology optimization strategies

Panel Discussion / Joint Q&A (15–20 min)

Theme: “From continuous sensitivity analysis to robust computational pipelines”
Topics may include: metric selection for descent, integrating turnpike insights into optimization loops, and reliable topology changes under discretization and regularization.