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Applications of the Force Density Method in optimum design of civil engineering structures

Optimization-Based Extensions of the Force Density Method for the Form Finding of Compressive Reticulated Shells

Matteo Bruggi, Politecnico di Milano

The Force Density Method (FDM) [1] is a well-established technique for the form finding of funicular and anti-funicular structures, as it transforms equilibrium equations into a linear system with respect to the unknown geometric parameters. However, the introduction of additional constraints generally leads to nonlinear formulations. In this context, gradient-based optimization methods can exploit the availability of analytical Jacobians to enforce equilibrium constraints while minimizing suitable objective functions. From this perspective, form finding can be formulated as an equilibrium-constrained structural optimization problem in which geometry is treated as a design variable [2].

This presentation discusses two recent extensions of the FDM for the design of compressive reticulated shells, where the inherent nonlinearities are addressed through mathematical programming.

The first extension generalizes the concept of force density to include bending moments and shear forces, enabling the form finding of gridshells with bending-resistant members. A constrained min-max optimization problem is formulated to determine shapes that minimize the maximum support reactions while satisfying equilibrium and geometric constraints [3].

The second extension incorporates compatibility and constitutive relationships within the FDM framework, enabling shape determination in a linear elastic setting. The resulting formulation simultaneously determines geometry and structural response, providing natural control over displacements and allowing the design of elastic no-tension configurations. Optimization is carried out by minimizing either the maximum support reactions or the structural compliance [4].

Through numerical examples, the presentation highlights how the inclusion of bending effects and elastic behaviour modifies the optimal forms obtained through traditional equilibrium-based approaches. Together, these developments exemplify how mathematical optimization can extend the Force Density Method into a unified framework for both equilibrium- and performance-driven form finding of compressive reticulated shells. Potential extensions of the proposed framework are finally discussed.

References

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