Numerical Form-Finding for Minimal Surfaces

Abstract

The presentation addresses the classical Plateau problem, namely, finding a minimal surface that spans a closed curve located above the boundary of a fixed region (design domain). To this end, two numerical methods are applied. The surface stress density method (SSDM) is well suited for triangulated, continuous surfaces because it assigns a stress-to-area ratio (stress density) to each surface triangle. The force density method (FDM) is a tool for discrete surfaces, known as gridshells, because it assigns a force-to-length ratio (force density) to each grid edge. The grid pattern is also assumed to be triangular.

The talk frames the problem in terms of the Dirichlet energy, which measures how much the surface stretches and distorts. Also, it provides an upper bound on the total surface area. The Euler–Lagrange equation for the discretized energy is equivalent to the SSDM and FDM equations weighted by the stress and force densities, respectively. With this, both methods effectively minimize the discretized Dirichlet energy on triangular meshes. Meshing of the design domain is restricted to Delaunay triangulations, which keep triangle shapes well-conditioned.

SSDM and FDM represent equilibrium shapes of tensioned surfaces under prestress. The choice of prestress values matters. The talk compares how these values influence the discretized Dirichlet energy and, consequently, the solution to the form-finding problem.