Wybrane zagadnienia optymalizacji inżynierskiej

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Plan prezentacji

1. Wprowadzenie

2. Trzy wybrane zagadnienia optymalizacji inżynierskiej

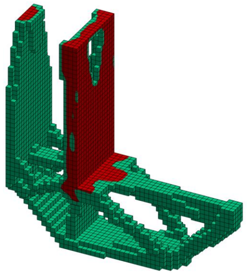
    2.1) Optymalizacja topologicznej konstrukcji

    2.2) Optymalne rozmieszczanie czujników

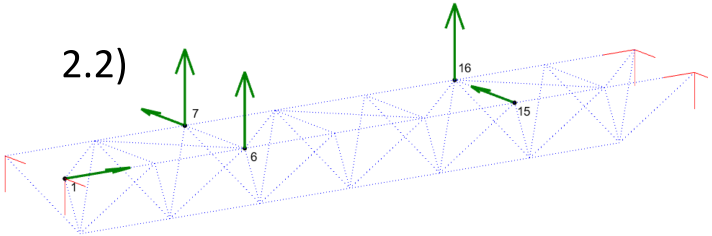
    2.3) Optymalne sterowanie konstrukcji o dynamicznie aktywowanych połączeniach

3. Wnioski końcowe

2.1)



2.2)



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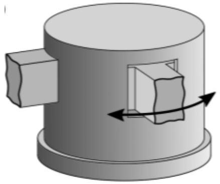
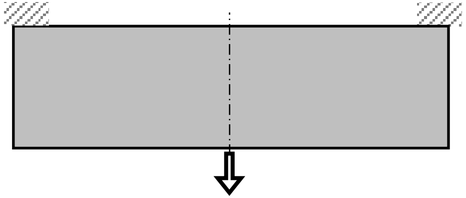


1. Introduction

I - Topology optimization

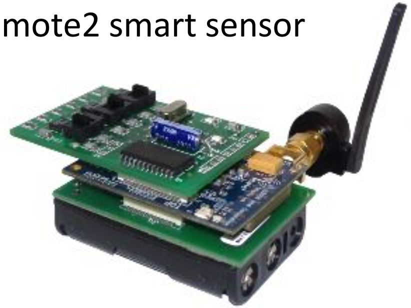


Design domain

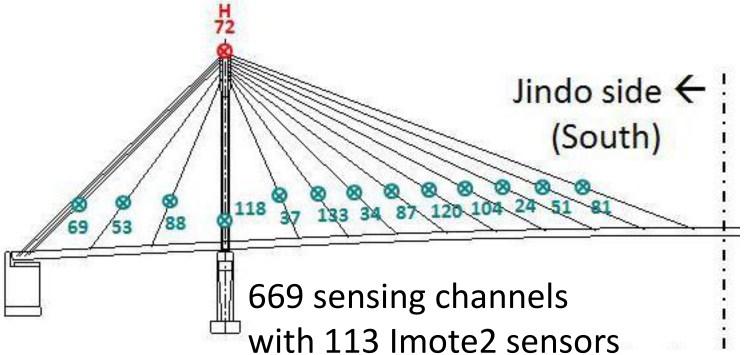
Optimal solution

II - Sensor placement

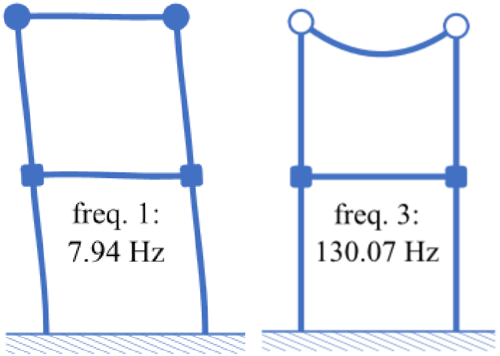
Imote2 smart sensor

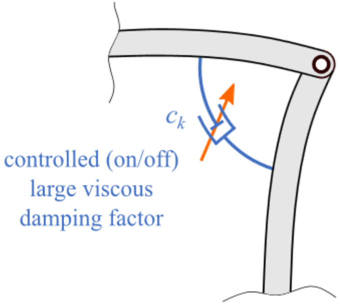


Smart Structures Technology Laboratory at the University of Illinois

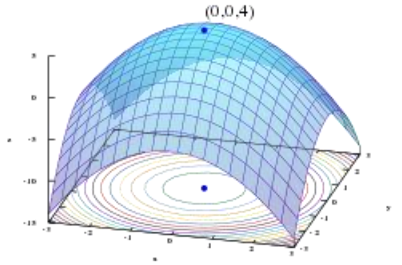


III – Semi-active control





1. Introduction



An optimization problem can be stated as follows

(ℙ0) find $\boldsymbol{x} = [x_1, x_2, \dots, x_M]^T$ which minimizes $f(\boldsymbol{x})$

subject to the constraints

$$g_i(\boldsymbol{x}) = 0, \quad i = 1,2, \dots, N$$

$$h_j(\boldsymbol{x}) \leq 0, \quad j = 1,2, \dots, P$$

where

$\boldsymbol{x}$ is an M- dimensional vector called the ***design vector***

$f(\boldsymbol{x})$ is termed the ***objective (cost) function***

$g_i(\boldsymbol{x})$ and $h_j(\boldsymbol{x})$ are known as ***equality*** and ***inequality constraints***, respectively

1. Introduction

'***No Free Lunch Theorem***' states that if any algorithm *A* outperforms another algorithm *B* in the search for an extremum of a cost function, then algorithm *B* will outperform *A* over other cost functions.

This means that the *universally best method does not exist*!

The main problem is how to find the better algorithms for a given particular type of problem.

D. H. Wolpert and W. G. Macready, "No free lunch theorems for optimization", *IEEE Transaction on Evolutionary Computation, 1,* 67-82 (1997).

1. Introduction

Classification of optimization algorithms

- algorithms
  - deterministic
    - linear programming
    - nonlinear programming
    - …
    - gradient-based
    - gradient-free
  - stochastic
    - heuristic
    - metaheuristic
      - population-based
      - trajectory-based

Deterministic algorithms follow a rigorous procedure and its path is repeatable

Stochastic algorithms always have some randomness

Dr. Piotr Tauzowski

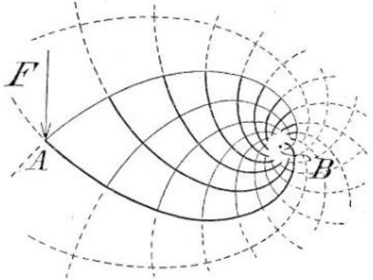
2.1. Structural topology optimization

2.1.1. Manufacturing

source:
https://www.arup.com

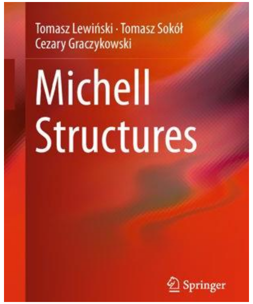
2.1.2. History of topology optimization

There is an additional new fact that topology optimization has started its career more than 100 years ago by Maxwell and only a few years later by Michell.



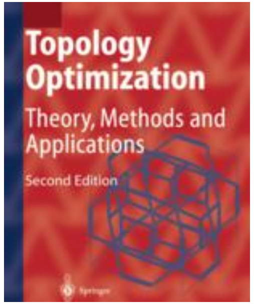
Michell, A.G.M. (1904) *The limits of economy of material in frame structures*. Phil. Mag. 8, 589-597

The classical solutions of the different type of plate or shell problems can be followed by the works of **Mróz, Prager** and **Shield**.



Lewiński, T., Sokół, T., Graczykowski, C. (2019) *Michell Structures,* Springer International Publishing AG

2.1.2. History of topology optimization - continued



Bendsoe and Sigmund (2004) *Topology Optimization: Theory, Methods, and Applications*

Minimum compliance problem

$$\min_{\mathbf{d}} f(\mathbf{d},\mathbf{u}) = \mathbf{u}^T\mathbf{K}(\mathbf{d})\mathbf{u}$$
$$s.t.\ g_1(\mathbf{d},\mathbf{u}) = \mathbf{K}(\mathbf{d})\mathbf{u} - \mathbf{p}$$
$$g_2(\mathbf{d}) = V(\mathbf{d}) - \overline{V}$$

If a rectangular design domain is considered and one uses square elements and a Q4 interpolation of displacements and element wise constant densities, a complete program can be written in **99 lines of Matlab code**

Two important issues that significantly influences the computational results are:

- the appearance of checkerboard and
- the mesh-dependency of results

2.1.3. Problem formulation

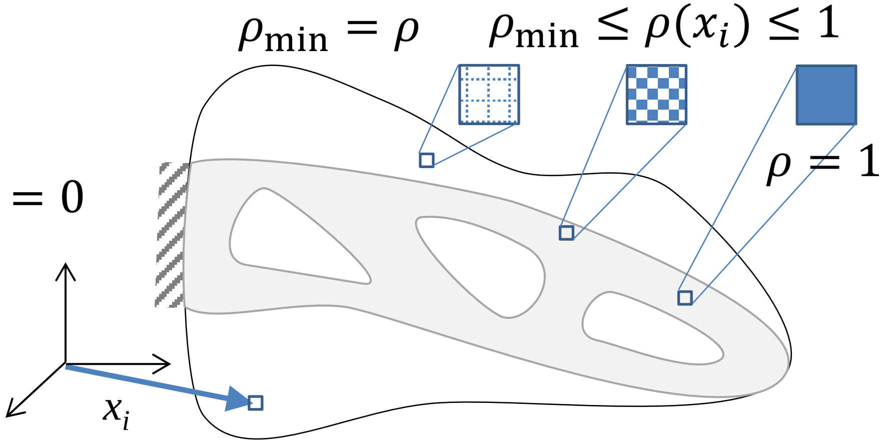
We are looking for minimum-weight design of structure made from elastoplastic material. Constraints are imposed on allowable stresses and density. This topology optimization problem can be expressed in the following form

minimize $\quad V = \int_V \rho(x_i) dV$

subject to $\quad \int_V \sigma_{ij} \delta\varepsilon_{ij} dV - \int_\Gamma f_i \delta u_i d\Gamma = 0$

$$d\sigma_{ij} = D^{\mathrm{ep}} d\varepsilon_{ij}$$

$$\sigma_{HMH} \leq \sigma_0$$



where V is the volume of the structure, σ_{ij} is the stress tensor, $\delta\varepsilon_{ij}$ is the virtual strain, f_i represents external loading, δu_i is the virtual displacement, D^{ep} – elastoplastic material stiffness tensor, σ_{HMH} is the Huber - von Mises - Hencky stress, σ_0 denotes the yield limit and finally $\rho(x_i)$ is density of the material distribution.

2.1.3. Spatial discretization

Then, discretized structural topology optimization investigated in this study can be expressed in the following form:

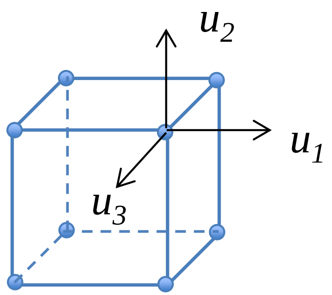
minimize $V = \boldsymbol{\rho}^T \boldsymbol{A}$

subject to $\mathbf{K}(\boldsymbol{\rho})\mathbf{u}(\boldsymbol{\rho}) - \mathbf{f} = \mathbf{0}$

$\boldsymbol{\sigma}_{\mathrm{HMH}} \leq \boldsymbol{\sigma}_0$

$\boldsymbol{\rho}_{\min} \leq \boldsymbol{\rho} \leq \mathbf{1}$

rectangular hexahedron finite element

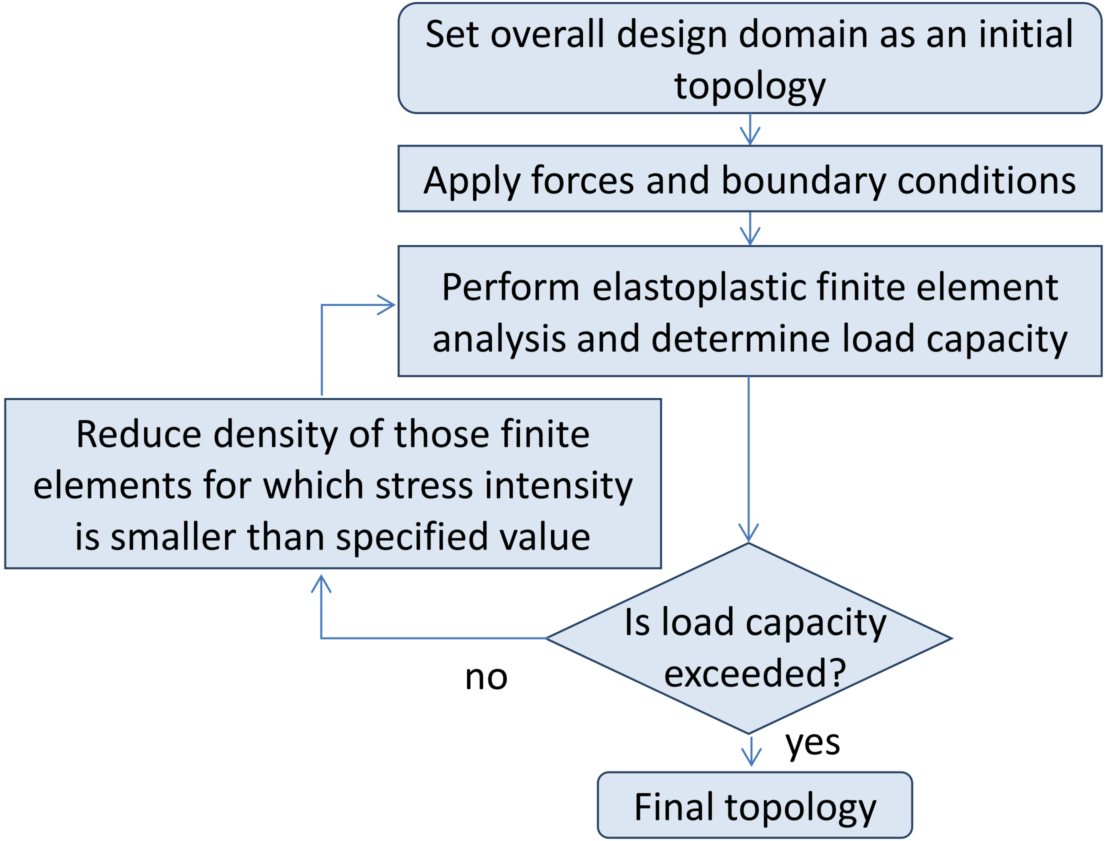


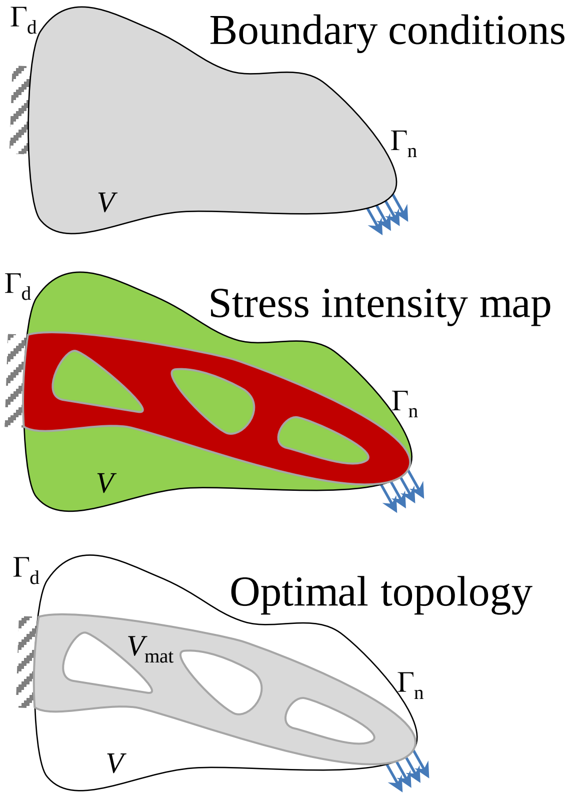
$$\mathbf{K}_e = \int_{V_e} \mathbf{B}^T \mathbf{D}^{ep} \mathbf{B} \, dV_e$$

$$\mathbf{K} = \mathcal{A}_{e=1}^{n_e} \mathbf{K}_e$$

where $\boldsymbol{A}$ is a vector representing area of individual finite element, $\mathbf{K}(\boldsymbol{\rho})$ denotes tangent stiffness matrix depending on the design variables $\boldsymbol{\rho}$, $\mathbf{u}(\boldsymbol{\rho})$ is displacement vector, $\mathbf{f}$ is external loading vector.

2.1.4. Proposed framework for stress-constrained optimization





2.1.4. Proposed framework for stress-constrained optimization

Algorithm 1. Stress intensity driven topology optimization

Step 1. Initialize design variables as a vector of ones $\boldsymbol{\rho}_e = \{1, 1, \ldots, 1\}$ and erased element list as an empty list $\mathcal{L} = \{\}$.

Step 2. Until load capacity is exceeded repeat ***Steps 3 to 7***.

Step 3. Solve nonlinear equilibrium equations for elastoplastic problem

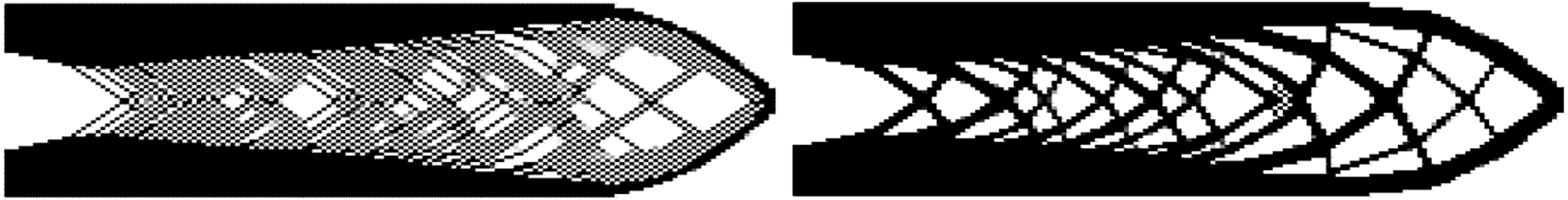
$$\mathbf{K}(\boldsymbol{\rho}_e)\mathbf{u}(\boldsymbol{\rho}_e) - \mathbf{f} = \mathbf{0}.$$

Step 4. Determine stress intensity vector calculated as average of equivalent von Mises stresses evaluated at each Gauss point, then normalize obtained value dividing it by yield limit

$$\bar{\sigma}_e = \frac{1}{n_g\, \sigma_0} \sum_{g=1}^{n_g} \sigma_{eq}^{g}, \qquad e = 1,2, \ldots, N .$$

2.1.4. Proposed framework for stress-constrained optimization

Step 5. Apply design filter to avoid checkerboard phenomenon.



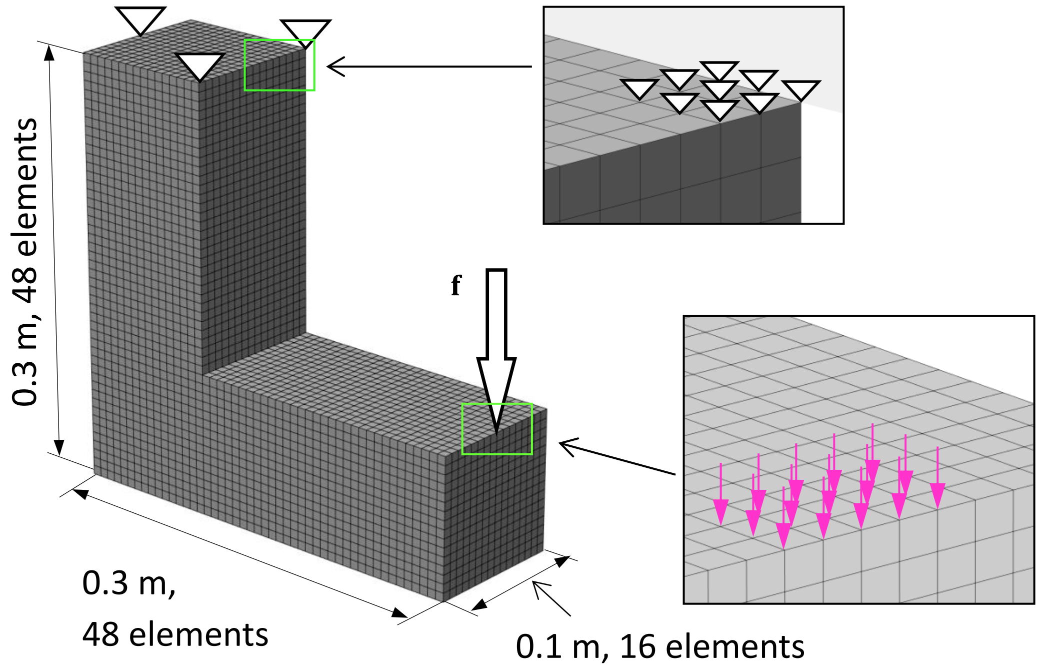
Step 6. Select *n* finite elements with smallest stresses and assign to their corresponding design variable values to $\rho_{\min}$. i.e. to the lower bound for design variables. Then, add the list of newly selected elements ℓ to the list of previously erased elements $\mathcal{L}^{\text{new}} = \{\mathcal{L}^{\text{old}};\ \ell\}$.

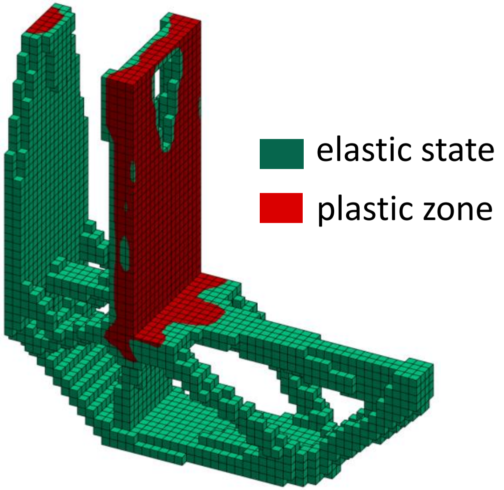
Step 7. Using current list of erased elements $\mathcal{L}$ update corresponding design variables applying the following iterative formula:

$$\rho_i^{\text{new}} = \max_{i \in \mathcal{L}}\left(\rho_{\min}, \bar{\sigma}_e{}^{p}\ \rho_i^{\text{old}}\right).$$

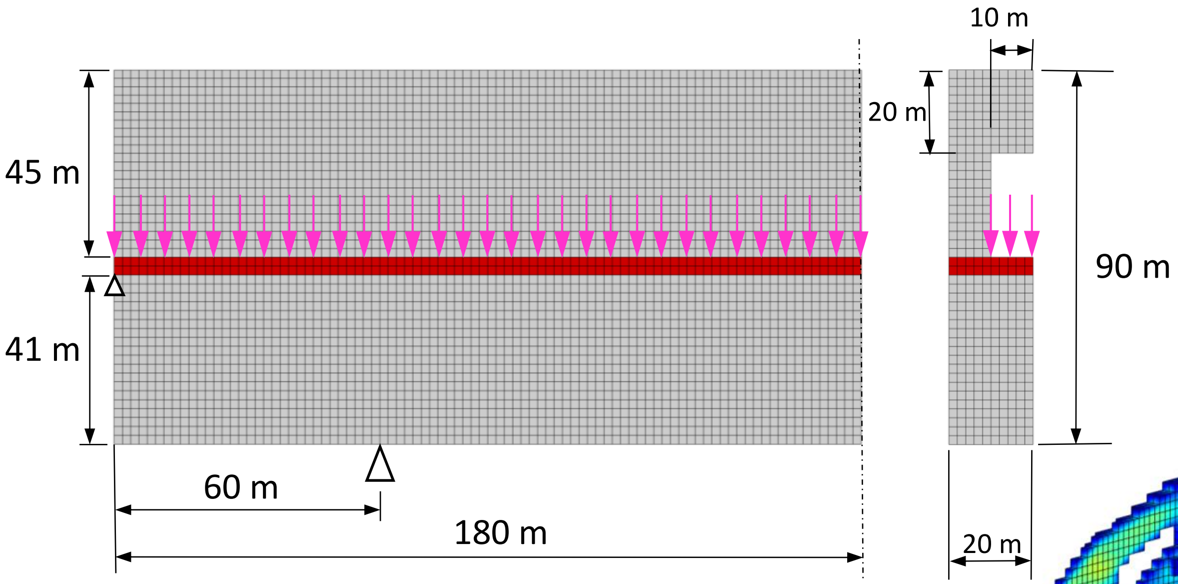
2.1.5. Illustrative examples – L-bracket design

The finite element model of the L-bracket. The mesh contains 20 480 finite elements.



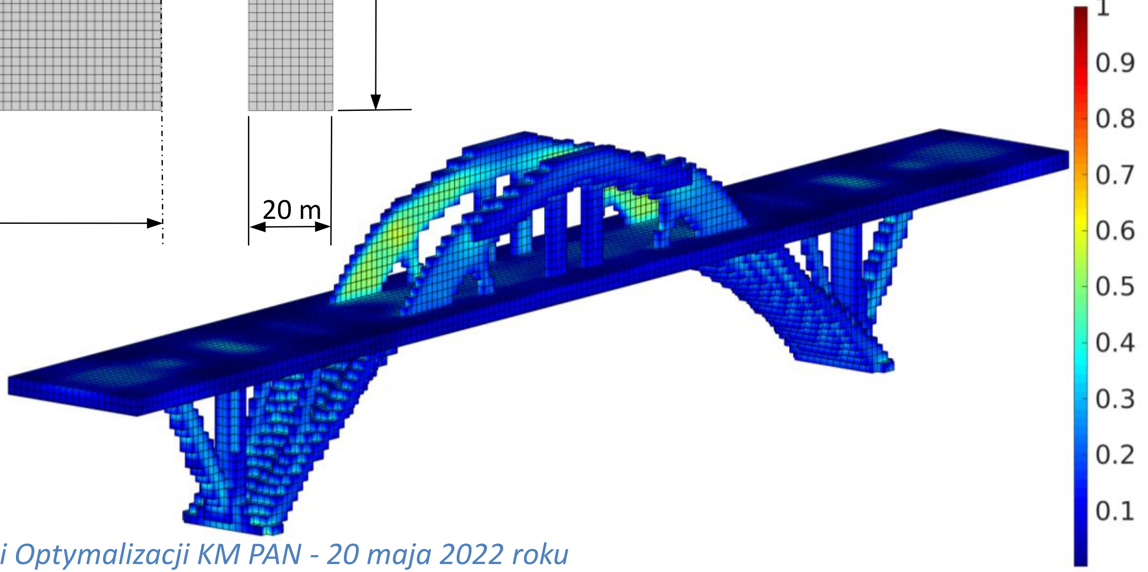


2.1.5. Illustrative examples – tied-arch bridge



design space of
tied-arch bridge

optimal topology of
tied arch bridge



Algorithm for finding optimal topology of structures under multiple / probabilistic loading

Step 1.

For $i = 1, 2, \ldots, Np$ (Number of multiple / probabilistic load cases)

Replace *i*-th load case with a set of three deterministic loadings for mean and two extreme values of random loading $\mathbf{f}_i \rightarrow \{ \mathbf{f}_{i,mean}, \mathbf{f}_{i,min}, \mathbf{f}_{i,max} \}$.

Step 2.

Next, determine all possible combinations of deterministic loads from Step 1.

For example $\mathbf{f}_j = \{ \mathbf{f}_{1,min}, \mathbf{f}_{2,max}, \ldots, \mathbf{f}_{Np,mean} \}$.

Step 3.

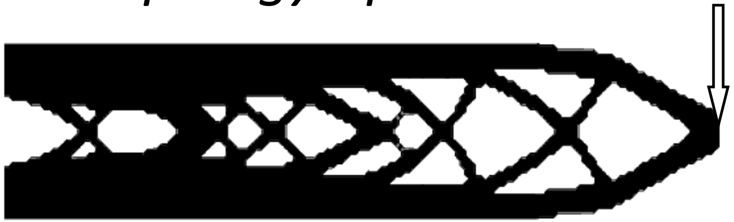
For each *j*-th load combination from Step 2. perform elastoplastic FE analysis to find stress intensity distribution within the structure under given combination.

Step 4.

Having stress intensity distribution for each load combination from Step 3. in every finite element determine maximal value of stress intensities over a set of load combinations.

This value of stress intensity will be used in an update formula for design variables.

2.1.6. Topology optimization under multiple load cases

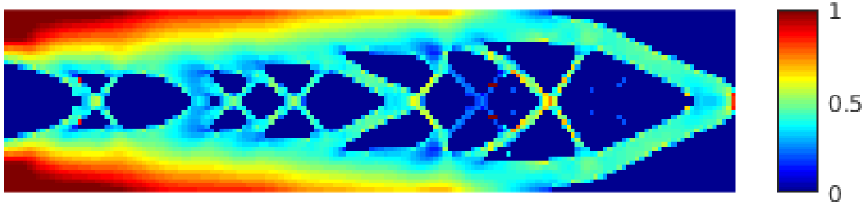


Optimal topology for single loading case
(α=0, iterations: 126, volume: 132.1 m^3)

Optimal topology for single loading case
($\alpha = \alpha_0$, iterations: 150, volume: 117.1 m^3)

Optimal topology for multiple loading case
(iterations: 123, volume: 134.2 m^3)

Envelope of optimal topologies for three single loading cases (volume: 154.3 m^3)



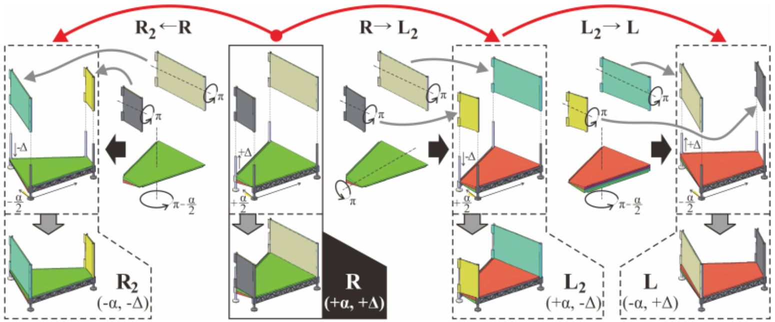
Stress intensity for multiple loading

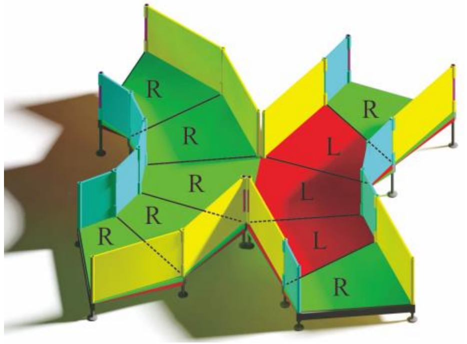
Blachowski, Tauzowski, Lógó - *Elasto-Plastic Topology Optimization Under Stochastic Loading Conditions*, EngOpt2018

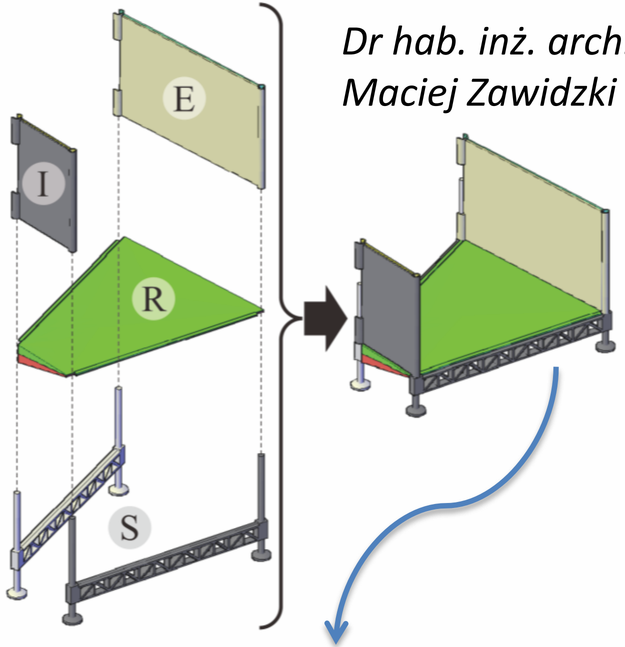
Modular ramp

Ramp-Z is an extremely modular system for creating free-form ramps

Dr hab. inż. arch. Maciej Zawidzki







Optimal topology for uniformly loaded clamped beam

2.1.7. Conclusions for part I

- A new optimal design method was presented in the field of elastoplasticity. The novelty is related to a computational procedure based on stress limited minimum volume design.

- Applied strategy provides optimal topologies comparable to ones obtained from other popular optimality criteria methods such as SIMP. However, contrary to SIMP in our method stresses are introduced directly into optimization process.

- Further extensions of the applied methodology are possible, including stress constrained topology optimization under multiple or probabilistic loading cases.

Dr. Andrzej Świercz

2.2. Optimal sensor placement

2.2.1. Introduction

„… Where should sensors be located in the system domain, so that the dynamic parameter estimates resulting from identification using the data obtained thereat have the smallest uncertainty ? …”

(Shah, Udwadia – *J. of Applied Mechanics, 1978)*

Smart Structures Technology Laboratory at University of Illinois

2.2.2. Constrained cardinality optimization problem

Topology or sensor placement optimization can be written as the following generic constrained optimization problem

$$(\mathbb{P}1) \qquad \underset{\boldsymbol{x}}{\text{minimize}} \quad \|\boldsymbol{x}\|_0$$

$$\text{subject to} \quad f(\boldsymbol{x}) \leq \lambda$$

$$\boldsymbol{x} = [x_1, x_2, \dots, x_M]^T \in \{0,1\}^M$$

where $\|\boldsymbol{x}\|_0$:=number of nonzero components of the vector $\boldsymbol{x}$

$f(\boldsymbol{x})$ is performance measure

λ is the threshold that specifies the accuracy requirements

$x_m = 1(0)$ indicates that the sensor is (not) selected

M is the number of sensors available

2.2.2. Constrained cardinality optimization problem

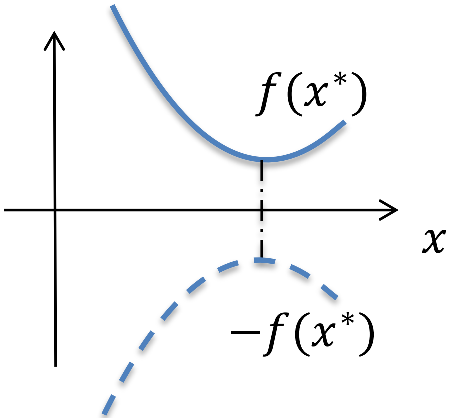
Naturally, the optimization problem in $(\mathbb{P}1)$ can also be casted as

$$(\mathbb{P}2) \qquad \underset{\boldsymbol{x}}{\text{minimize}} \quad f(\boldsymbol{x})$$

$$\text{subject to} \quad \|\boldsymbol{x}\|_0 = K$$

$$\boldsymbol{x} \in \{0,1\}^M$$

where K is the desired number of sensors



$(\mathbb{P}1)$ is a non-convex optimization problem with a non-convex cost function. The non-convex Boolean constraint incurs a combinatorial search over all the 2^M possible combinations.

This number for the optimization problem of the form $(\mathbb{P}2)$ is $\binom{K}{M}$.

2.2.2. Constrained cardinality optimization problem

To simplify the problem, standard convex relaxations are used. The ℓ_0-(quasi) norm in $(\mathbb{P}1)$ is relaxed to the ℓ_1-norm, and the Boolean constraint is relaxed to the box constraint $[0,1]^M$. As a result, the following relaxed sensor selection problem is obtained

$$(\mathbb{P}3) \qquad \underset{\boldsymbol{x}}{\text{minimize}} \quad \|\boldsymbol{x}\|_1$$

$$\text{subject to} \quad f(\boldsymbol{x}) \le \lambda$$

$$\boldsymbol{x} \in [0,1]^M$$

where $\|\boldsymbol{x}\|_1$:=sum of absolute values of the vector $\boldsymbol{x}$ components

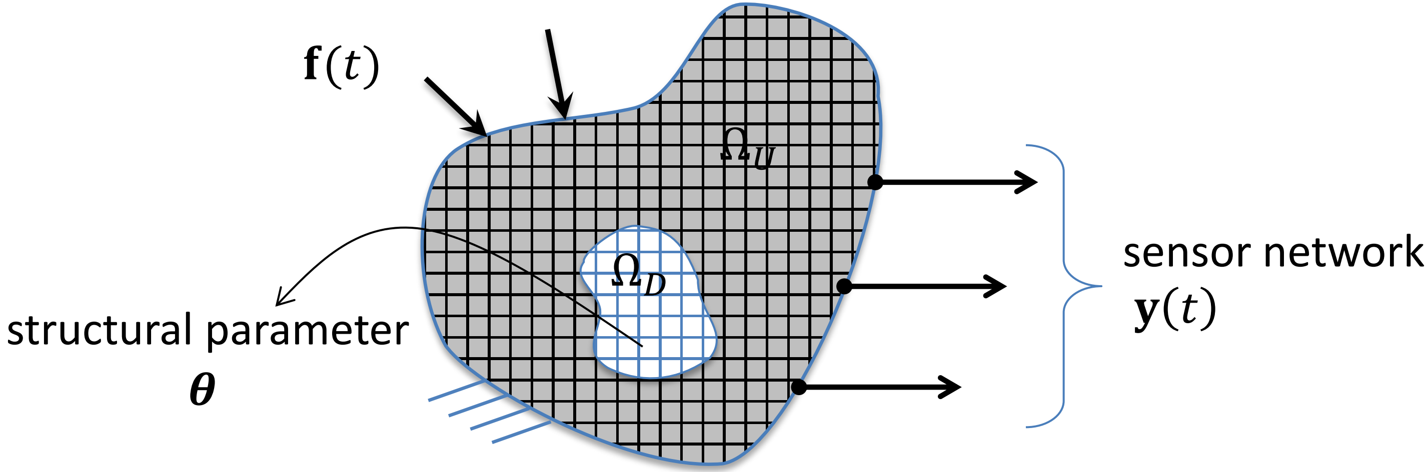
2.2.3. Sensor placement as a discrete optimization problem

Equations of motion

$$\mathbf{M}(\boldsymbol{\theta})\ddot{\mathbf{q}}(\boldsymbol{\theta},t)+\mathbf{C}(\boldsymbol{\theta})\dot{\mathbf{q}}(\boldsymbol{\theta},t)+\mathbf{K}(\boldsymbol{\theta})\mathbf{q}(\boldsymbol{\theta},t)=\mathbf{f}(t)$$

Observation equation

$$\mathbf{y}(t)=\mathbf{C}_a\ddot{\mathbf{q}}(\boldsymbol{\theta},t)+\mathbf{C}_v\dot{\mathbf{q}}(\boldsymbol{\theta},t)+\mathbf{C}_d\mathbf{q}(\boldsymbol{\theta},t)$$



2.2.3. Estimation error and metrics for optimal sensor placement

The structural response vector in modal coordinates

$$\mathbf{q}(t) = \boldsymbol{\Phi}\boldsymbol{\eta}(t)$$

where $\mathbf{q} \in \mathbb{R}^{n_d}$, $\boldsymbol{\Phi} \in \mathbb{R}^{n_d \times n_m}$ and $\boldsymbol{\eta} \in \mathbb{R}^{n_m}$. Indices n_d and n_m denote the number of DOFs and the number of modes, respectively.

The observation equation in modal coordinates

$$\mathbf{y}_s(t) = \boldsymbol{\Phi}_s\boldsymbol{\eta}(t) + \mathbf{w}(t)$$

measurement errors

where $\boldsymbol{y}_s \in \mathbb{R}^{n_s}$, $\boldsymbol{\Phi}_s \in \mathbb{R}^{n_s \times n_m}$ and $\mathbf{w} \in \mathbb{R}^{n_s}$. Matrix $\boldsymbol{\Phi}_s$ denotes measured components of the modal matrix $\boldsymbol{\Phi}$ and index n_s is the number of sensors.

2.2.3. Estimation error and metrics for optimal sensor placement

Assuming that $n_s \geq n_m$, the least square estimate of modal coordinates can be determined as follows

$$\tilde{\boldsymbol{\eta}}(t) = \Big(\underbrace{\boldsymbol{\Phi}_s^{\mathrm{T}}\boldsymbol{\Phi}_s}\Big)^{-1}\boldsymbol{\Phi}_s^{\mathrm{T}}\mathbf{y}_s(t) = \boldsymbol{\Phi}_s^{+}\mathbf{y}_s(t)$$

Fisher information matrix (FIM)

An estimate of the structural response can be determined as follows

$$\tilde{\mathbf{q}}(t) = \boldsymbol{\Phi}\tilde{\boldsymbol{\eta}}(t) = \boldsymbol{\Phi}\boldsymbol{\Phi}_s^{+}\mathbf{y}_s(t)$$

The estimation error

$$\boldsymbol{e}(\mathrm{t}) = \tilde{\mathbf{q}}(t) - \mathbf{q}(t) = \boldsymbol{\Phi}\boldsymbol{\Phi}_s^{+}\mathbf{w}(t)$$

2.2.3. Estimation error and metrics for optimal sensor placement

If the components of the measurement noise have zero mean $E\{\mathbf{w}(t)\} = \mathbf{0}$ and are uncorrelated $E\{\mathbf{w}(t)\mathbf{w}(t)^T\} = \sigma_w^2\mathbf{I}$, the covariance matrix of the estimate error takes the following form

$$E\{\boldsymbol{e}(\mathrm{t})\boldsymbol{e}(\mathrm{t})^{\mathrm{T}}\} = \sigma_w^2\boldsymbol{\Phi}\boldsymbol{\Phi}_s^{+}(\boldsymbol{\Phi}_s^{+})^{\mathrm{T}}\boldsymbol{\Phi}^{\mathrm{T}}$$

noise variance specific to a selected type of sensors

To compare two different sensor configurations, two metrics are frequently used in the literature:

$$\sigma_{e,\mathrm{avg}}^2 = \frac{1}{n_s}\mathrm{tr}\left(E\{\boldsymbol{e}(\mathrm{t})\boldsymbol{e}(\mathrm{t})^{\mathrm{T}}\}\right)$$

$$\sigma_{e,\mathrm{max}}^2 = \max\left\{\mathrm{diag}\left(E\{\boldsymbol{e}(\mathrm{t})\boldsymbol{e}(\mathrm{t})^{\mathrm{T}}\}\right)\right\}$$

2.2.4. Combinatorial methods for optimal sensor placement

The number of ways in which n_s sensor positions can be chosen from among n_d candidate locations is given by the well-known formula

$$\binom{n_d}{n_s} = \frac{n_d!}{n_s!\,(n_d - n_s)!}$$

Effective Independence procedure (*Kammer 1991*)

initialize: select n_m *// number of modes to be monitored*

assign $n_s \leftarrow n_d$ *// assume sensors present at all candidate locations*

while $n_s > n_m$

form Fisher Information Matrix $\mathbf{FIM} = \boldsymbol{\Phi}_s^{\mathrm{T}} \boldsymbol{\Phi}_s$

determine contribution of each measurement location to the rank of $\mathbf{FIM} = \mathrm{tr}\left(\boldsymbol{\Phi}_s \mathbf{FIM}^{-1} \boldsymbol{\Phi}_s^{\mathrm{T}}\right)$

remove the degree of freedom that contributes least to the independence of chosen modes

return a set of remaining co-ordinates S

2.2.5. Proposed approach for optimal sensor placement

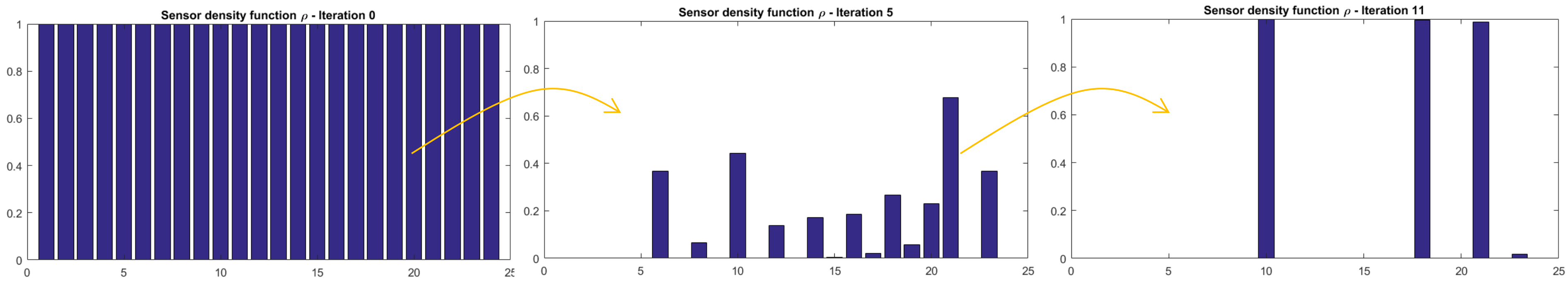
For the purpose of continuous optimization we introduce the function of sensor density $\boldsymbol{\rho}(\boldsymbol{x})$, which takes values between 0 and 1.

Then, instead of removing individual rows from the full modal matrix we multiply it by the value of the sensor density function at a given location $\boldsymbol{x}$

$$\boldsymbol{\Phi}_s = \mathrm{diag}\big(\boldsymbol{\rho}(\boldsymbol{x})\big)\boldsymbol{\Phi}$$

Sensor density function — $\boldsymbol{\rho}(\boldsymbol{x})$

Original mode shape matrix — $\boldsymbol{\Phi}$



2.2.5. Effective algorithm for optimal sensor placement

initialize: select n_m

assign $n_s \leftarrow n_d,\ \boldsymbol{\rho}^{(0)} = 1$

form Fisher Information Matrix $\mathbf{FIM} = \boldsymbol{\Phi}_s^{\mathrm{T}} \boldsymbol{\Phi}_s$

determine sensor density function $\boldsymbol{\rho}^{(1)} = \mathrm{diag}\left(\boldsymbol{\Phi}_s \mathbf{FIM}^{-1} \boldsymbol{\Phi}_s^{\mathrm{T}}\right)$

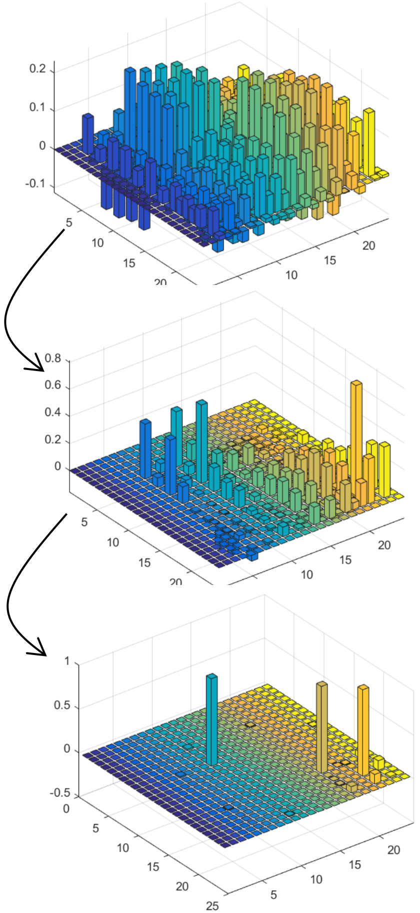
while $\left\|\boldsymbol{\rho}^{(i+1)} - \boldsymbol{\rho}^{(i)}\right\| > \varepsilon$

form Fisher Information Matrix $\mathbf{FIM} = \boldsymbol{\Phi}_s^{\mathrm{T}} \boldsymbol{\Phi}_s$

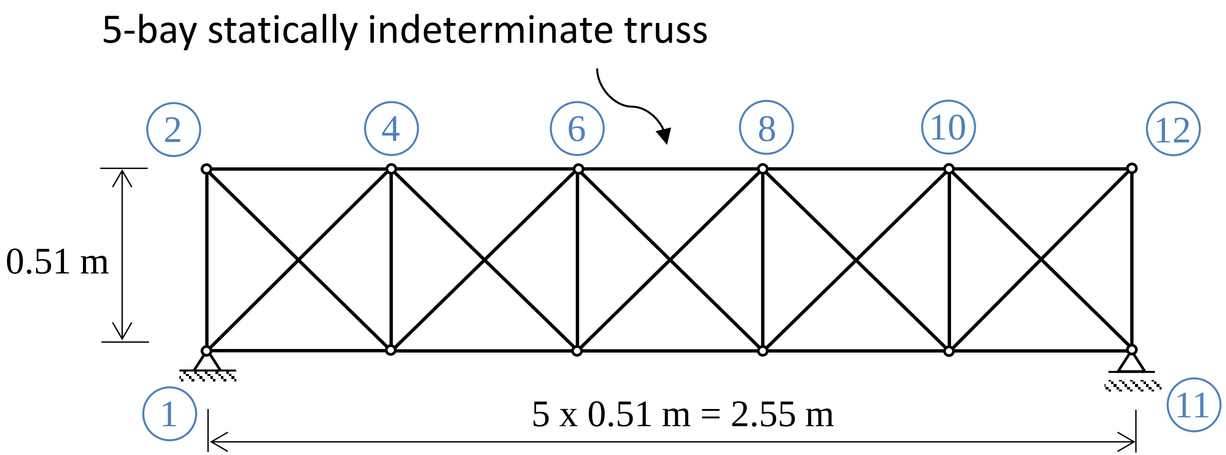
save values on diagonal of idempotent matrix $\mathbf{d}^{(i+1)} = \mathrm{diag}\left(\boldsymbol{\Phi}_s \mathbf{FIM}^{-1} \boldsymbol{\Phi}_s^{\mathrm{T}}\right)$

update values of sensor density function $\boldsymbol{\rho}^{(i+1)} = {\mathbf{d}^{(i+1)}}\big/{\max\limits_{\boldsymbol{x}}\left(\mathbf{d}^{(i+1)}\right)}$

return recent value of the sensor density function

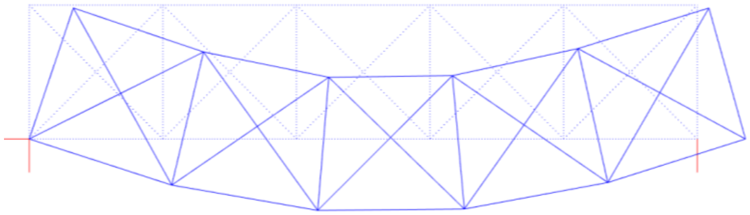


2.2.6. Illustrative example – simple truss



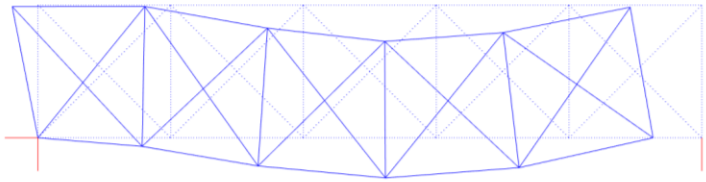
The steel elements with a length of L_x=L_y=51 cm have the cross section areas A = 10^{-4} m^2, mass density ρ = 7850 kg/m^3 and Young's modulus E = 200 GPa.

2.2.6. Simple truss – mode shapes



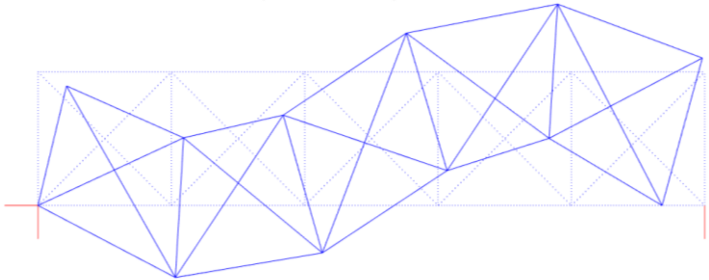
1st bending mode

$\omega_1 = 919.0 \text{ rad/s}$



1st longitudinal mode

$\omega_2 = 1745.2 \text{ rad/s}$



2nd bending mode

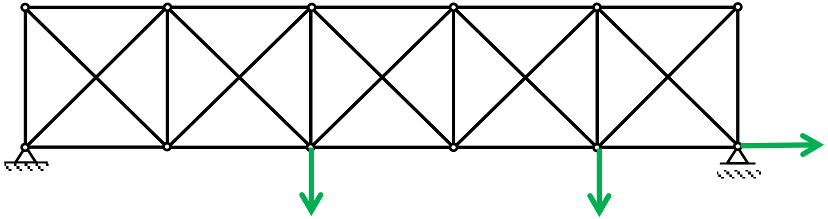
$\omega_3 = 3044.7 \text{ rad/s}$

n_d = 24 DOFs

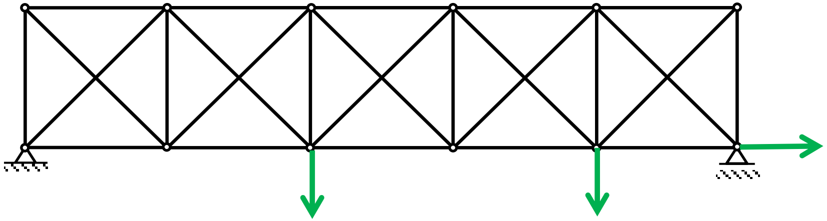
n_s = 3 sensors

$$\frac{n_d!}{n_s!(n_d - n_s)!} = \frac{24!}{3!\,21!} = 2024 \text{ combinations}$$

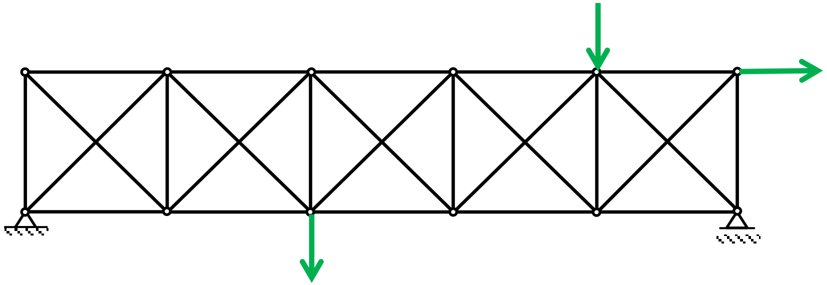
2.2.6. Simple truss – optimal sensor placement



C1) Sensor configuration obtained by Effective Independence algorithm



C2) Sensor configuration obtained by convex relaxation based approach

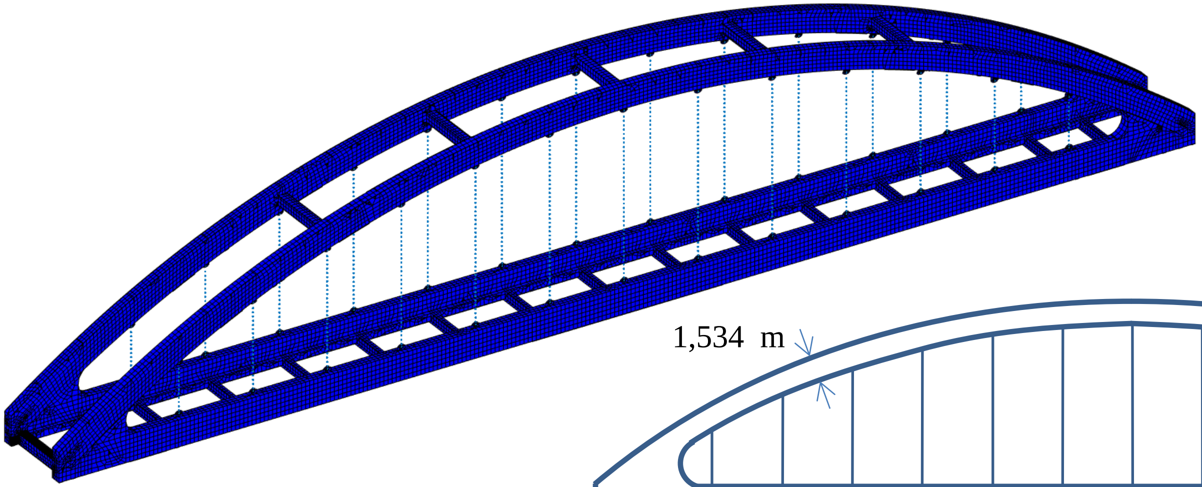


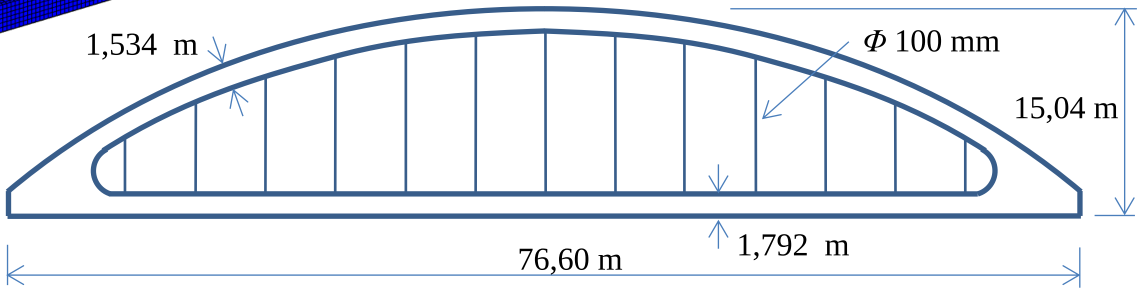
C3) Optimal location from full enumeration

No.	Sensor location	Determinant of FIM
C1)	10, 18, 21	2.0491
C2)	10, 18, 21	2.0491
C3)	10, 20, 23	2.0569

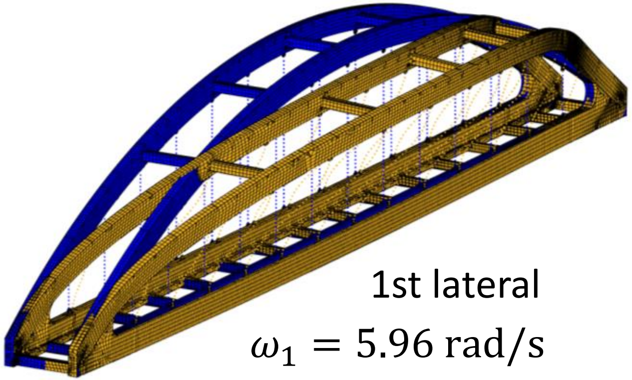
2.2.7. Illustrative example – arch bridge

The second example is a real tied-arch bridge located in Poland. The bridge's main span is 76.6-m long and consists of an arch box girder supporting the bridge deck using 13 vertical members (hangers). The arch and hangers are made of steel with the following material properties:
E = 210 GPa, ν = 0.2, ρ = 7850 kg/m^3.

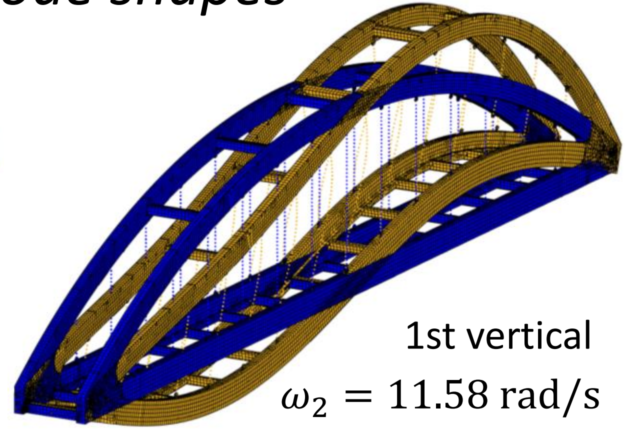




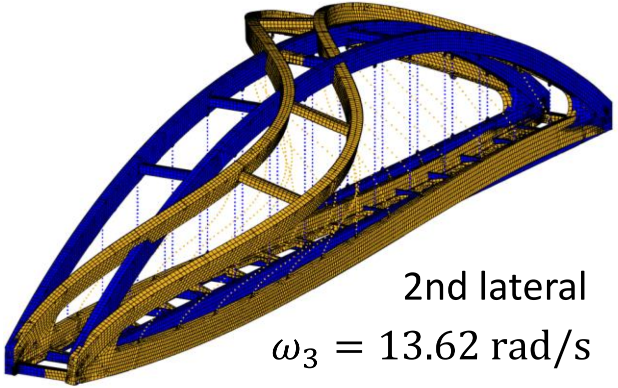
2.2.7. Arch bridge – mode shapes



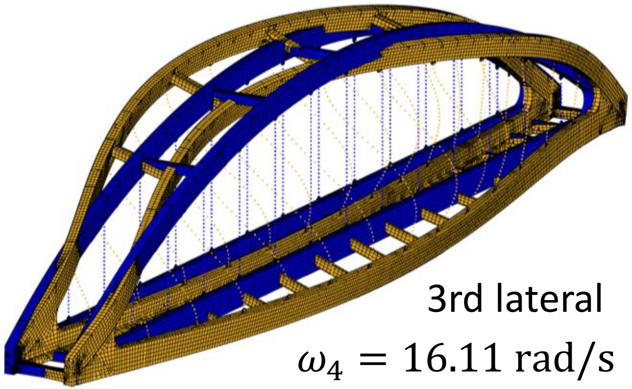
1st lateral
$\omega_1 = 5.96 \text{ rad/s}$



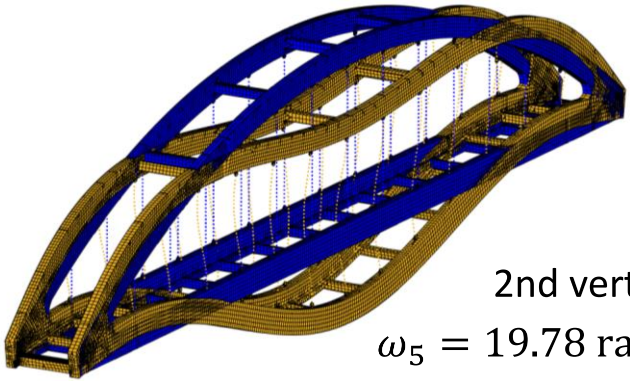
1st vertical
$\omega_2 = 11.58 \text{ rad/s}$



2nd lateral
$\omega_3 = 13.62 \text{ rad/s}$



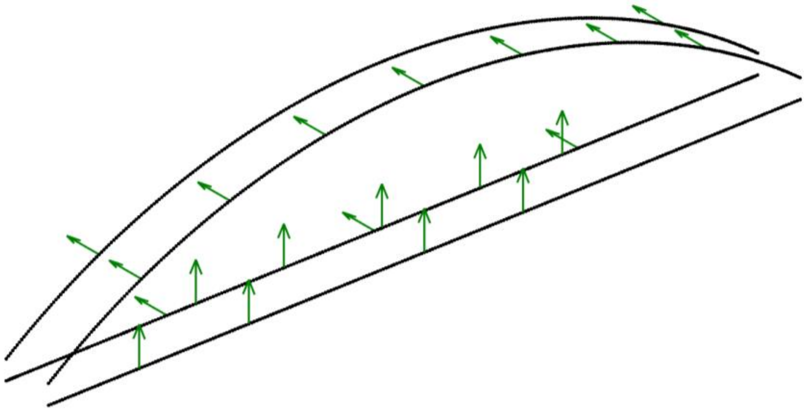
3rd lateral
$\omega_4 = 16.11 \text{ rad/s}$



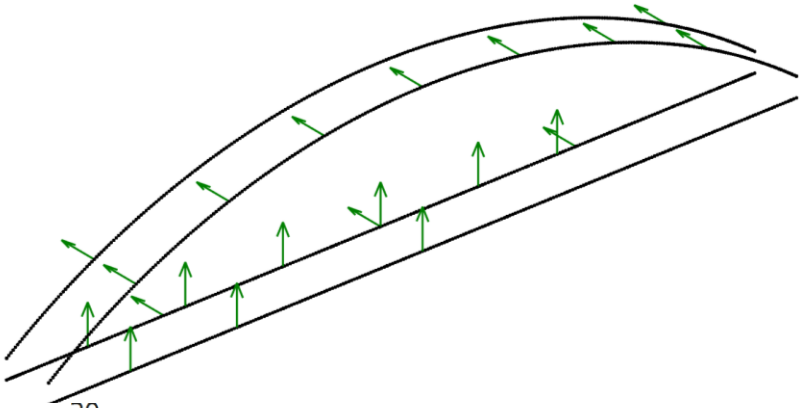
2nd vertical
$\omega_5 = 19.78 \text{ rad/s}$

2.2.7. Arch bridge – optimal sensor placement

Effective independence

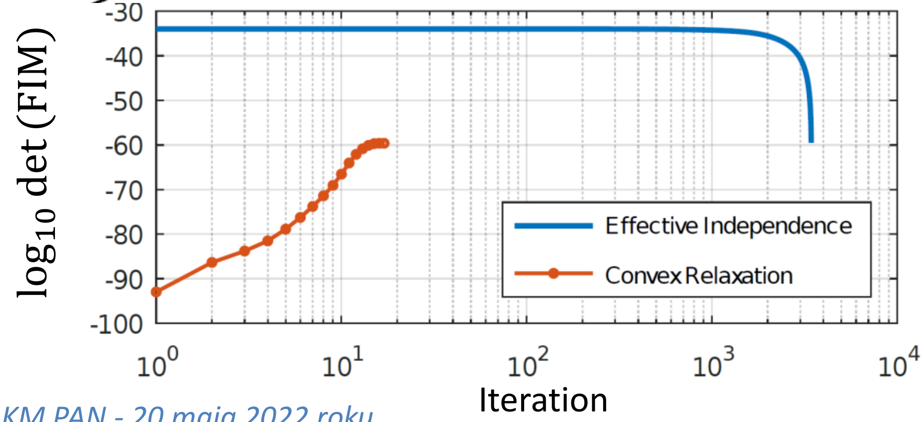


Convex relaxation



For determining 21 sensor locations

3490 computational steps were required



2.2.8. Conclusions for part II

- An efficient method for dense sensor network deployment over large structures was presented. The proposed approach is based on an analogy between sensor placement and topology optimization.
- Utilizing this observation a sensor density function has been introduced, which allowed to approach the optimal solution in an iterative way instead of sequential removal of individual less significant degrees of freedom.
- The effectiveness of the proposed methodology has been demonstrated using two case studies: a simple 5-bay truss bridge and a real scale tied-arch bridge.

Mr. Mariusz Ostrowski

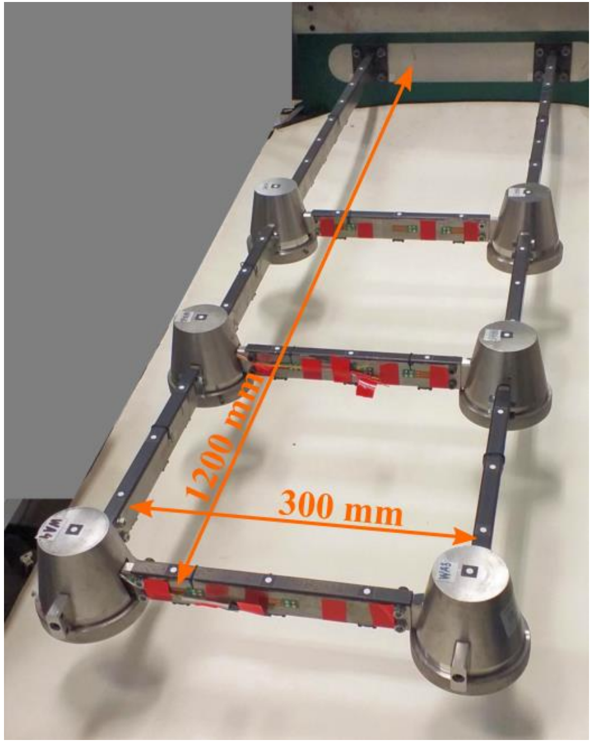
2.3. Semi-active control

2.3.1. Semi-active control of lockable joints

In this study, a novel modal control strategy by means of semi-actively lockable joints is proposed.

The control strategy is an extension of the Prestress-Accumulation Release (PAR) technique; however, it introduces also new concepts that increase the efficiency of the overall control system. Contrary to the PAR, the proposed method requires measurement of both strains in the vicinity of the semi-active joints and translational velocities that provide global information about system behavior.

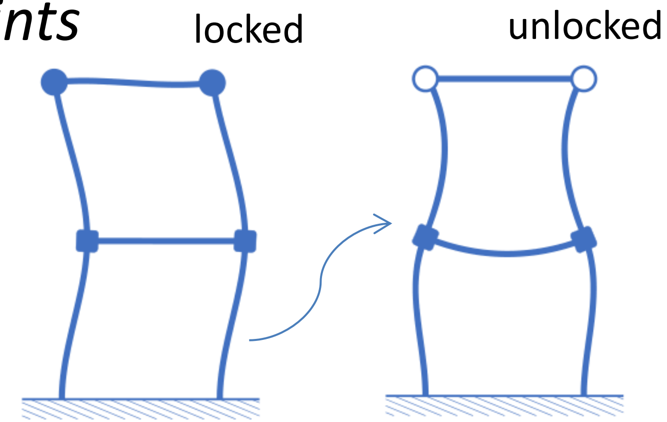
The benefit from this higher complexity of the control system is its better performance compared to the PAR.



2.3.1. Semi-active control of lockable joints

We are looking for control strategy that allows to switch from vibrations in one mode into the another one.

This goal is going to be achieved by applying the concept of lockable joint.

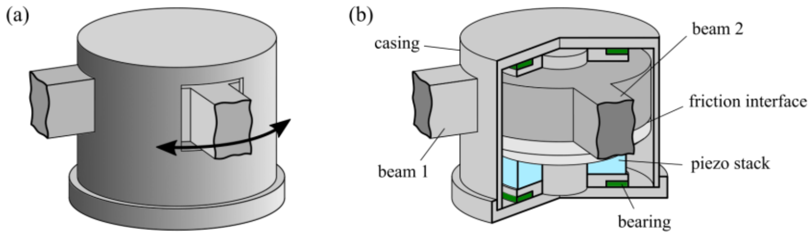


In the proposed methodology the lockable joint can take one of the two opposite states:

(1) fully locked – friction is sufficiently large to lock any rotation between rotational DOFs,

(2) fully unlocked – no any friction between rotational DOFs.

Fig. Lockable joint semi-actively controlled with piezo stack: (a) general view, (b) concept of an implementation of individual components



2.3.2. Simplified model of the lockable joint

System dynamics:

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{B}\mathbf{f}(t) + \mathbf{d}(t)$$

where

$$\mathbf{f}(t) = [f_1(t) \quad \cdots \quad f_k(t) \quad \cdots \quad f_{N_k}(t)]^{\mathrm{T}}$$

Appling viscous model lockable joint can be described by equation below:

$$f_k(t) = -u_k c_{\max}\left(\dot{q}_i(t) - \dot{q}_j(t)\right) = -\underbrace{u_k c_{\max}}_{c_k} \Delta\dot{q}_k(t)$$

where i and j denote DOFs involved in kth lockable joint, $c_{\max}$ is large viscous damping.

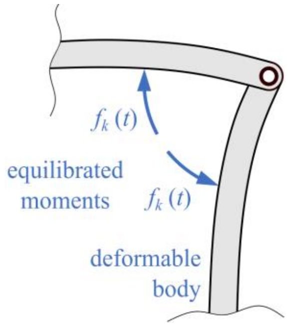


Fig. Pair of self-equilibrated bending moments at k-th locked joint

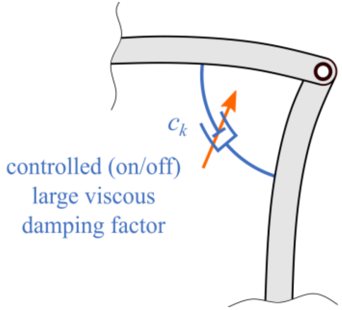


Fig. Viscous damper equivalent to self-equilibrated moments $f_k(t)$

Substituting viscous model of the lockable joint into equation of motion the dynamics of structure is described by the following equations:

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \left(\mathbf{C}_0 + \sum_{k=1}^{N_k} u_k \mathbf{C}_k\right)\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{d}(t),$$

where $u_k \in \{0,1\}$, $\mathbf{C}_k = c_{\max}\mathbf{L}_k{}^{\mathrm{T}}\mathbf{L}_k$, and $\mathbf{L}_k = [0 \quad \cdots \quad 1 \quad \cdots \quad 0 \quad \cdots \quad -1 \quad \cdots \quad 0]$

Using undamped mode shapes (for all joints unlocked) $\mathbf{q}(t) = \mathbf{\Phi}\boldsymbol{\eta}(t)$

where $\mathbf{\Phi} = [\boldsymbol{\phi}_1 \quad \boldsymbol{\phi}_2 \quad \ldots \quad \boldsymbol{\phi}_m]$ and $(\mathbf{K} - \omega_m{}^2\mathbf{M})\boldsymbol{\phi}_m = \mathbf{0}$, the system dynamics can be represented in modal domain in the following way

$$\ddot{\boldsymbol{\eta}}(t) + \left(\mathbf{\Gamma}_0 + \sum_{k=1}^{N_k} u_k \mathbf{\Gamma}_k\right)\dot{\boldsymbol{\eta}}(t) + \mathbf{\Omega}^2\boldsymbol{\eta}(t) = \mathbf{\Phi}^{\mathrm{T}}\mathbf{d}(t)$$

for m-th mode:

$$\ddot{\eta}_m(t) + 2\zeta_m\omega_m\dot{\eta}_m(t) + \omega_m^2\eta_m(t) = -\sum_{k=1}^{N_k}\sum_{n=1}^{N_m} u_k\gamma_{kmn}\dot{\eta}_n(t) + \sum_{i=1}^{N_m}\phi_i^{(m)}d_i(t)$$

Total mechanical energy is the sum of modal energies:

$$E(t) = \frac{1}{2}\left(\dot{\mathbf{q}}^{\mathrm{T}}(t)\mathbf{M}\dot{\mathbf{q}}(t)+\mathbf{q}^{\mathrm{T}}(t)\mathbf{K}\mathbf{q}(t)\right)= \sum_{m=1}^{N_m}\frac{1}{2}\left(\dot{\eta}_m^2(t) + \omega_m^2\eta_m^2(t)\right) = \sum_{m=1}^{N_m} E_m(t)$$

Total increment of mth modal energy can be calculated as follows:

$$\dot{E}_m(t) = \dot{\eta}_m(t)(\ddot{\eta}_m(t) + \omega_m^2\eta_m(t))$$

$$= \underbrace{-\dot{\eta}_m(t)\sum_{k=1}^{N_k}\sum_{n=1}^{N_m} u_k\gamma_{kmn}\dot{\eta}_n(t)}_{\dot{W}_m(t,\mathbf{u})} + \underbrace{\dot{\eta}_m(t)\sum_{i=1}^{N_m}\phi_i^{(m)}d_i(t)}_{\dot{W}_{\mathrm{ext}}(t)} - \underbrace{2\zeta_m\omega_m\dot{\eta}_m^2(t)}_{\dot{E}_{\mathrm{loss}}(t)}$$

W_m - energy transferred to m-th vibration mode from other ones

2.3.3. Instantaneous optimal control

$$\dot{W}_m(t,\mathbf{u}) = -\dot{\eta}_m(t)\sum_{k=1}^{N_k}\sum_{n=1}^{N_m} u_k\gamma_{kmn}\dot{\eta}_n(t) = \sum_{k=1}^{N_k}\underbrace{\dot{\eta}_m(t)\Delta\phi_k^{(m)}}_{\Delta\dot{q}_k^{(m)}}\underbrace{\left(-u_k c_{\max}\Delta\dot{q}_k(t)\right)}_{f_k(t)}$$

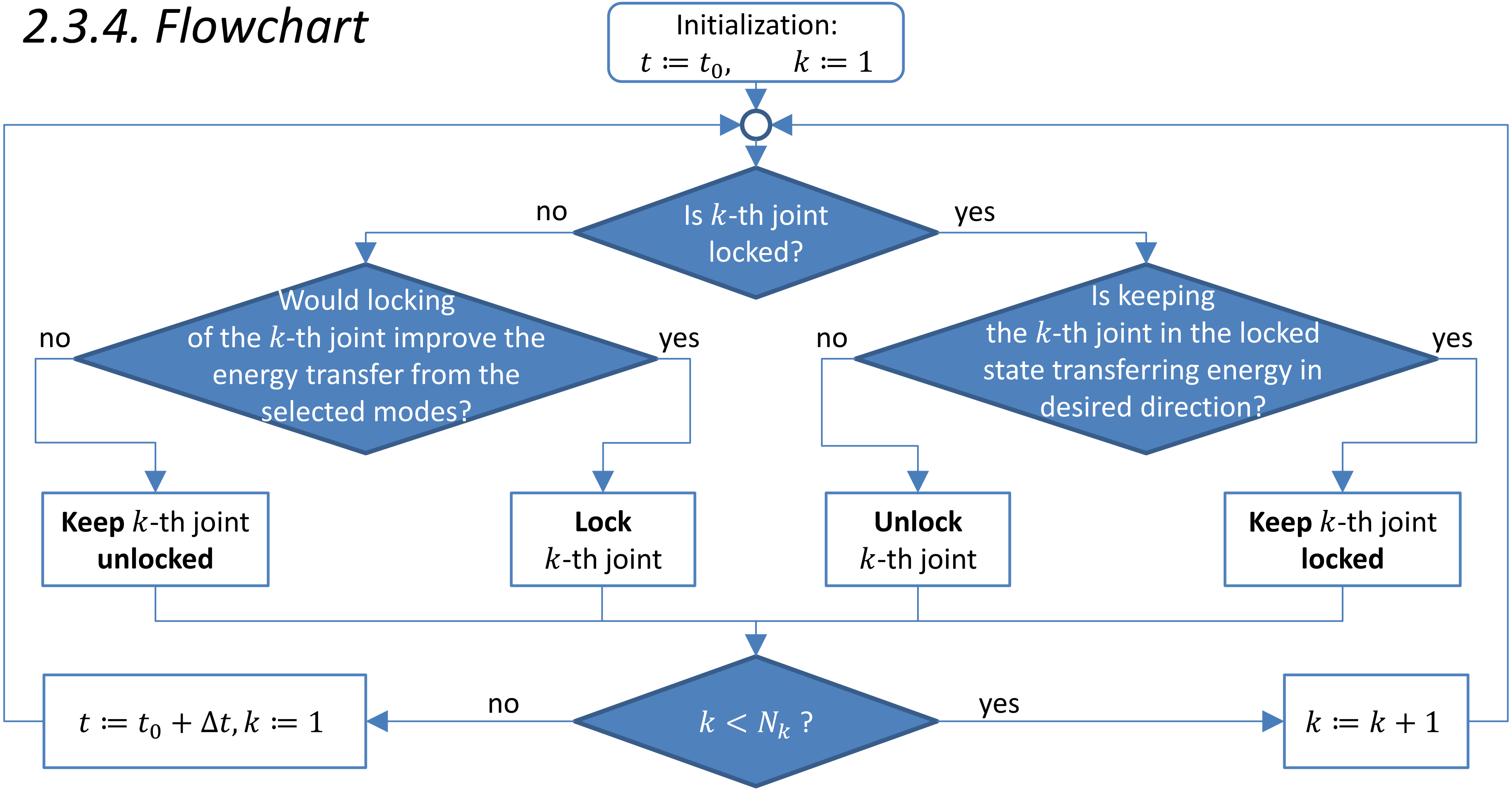
Cumulative energy transfer from the pth mode is equal to the sum of energy transferred by individual lockable joints:

$$\sum_{p=1}^{N_p}\alpha_p\dot{W}_p(t,\mathbf{u}) = -\sum_{p=1}^{N_p}\alpha_p\dot{\eta}_p(t)\sum_{k=1}^{N_k}\sum_{n=1}^{N_m}u_k\gamma_{kpn}\dot{\eta}_n(t) = \sum_{k=1}^{N_k}\sum_{p=1}^{N_p}\alpha_p\dot{W}_{pk}(t,u_k)$$

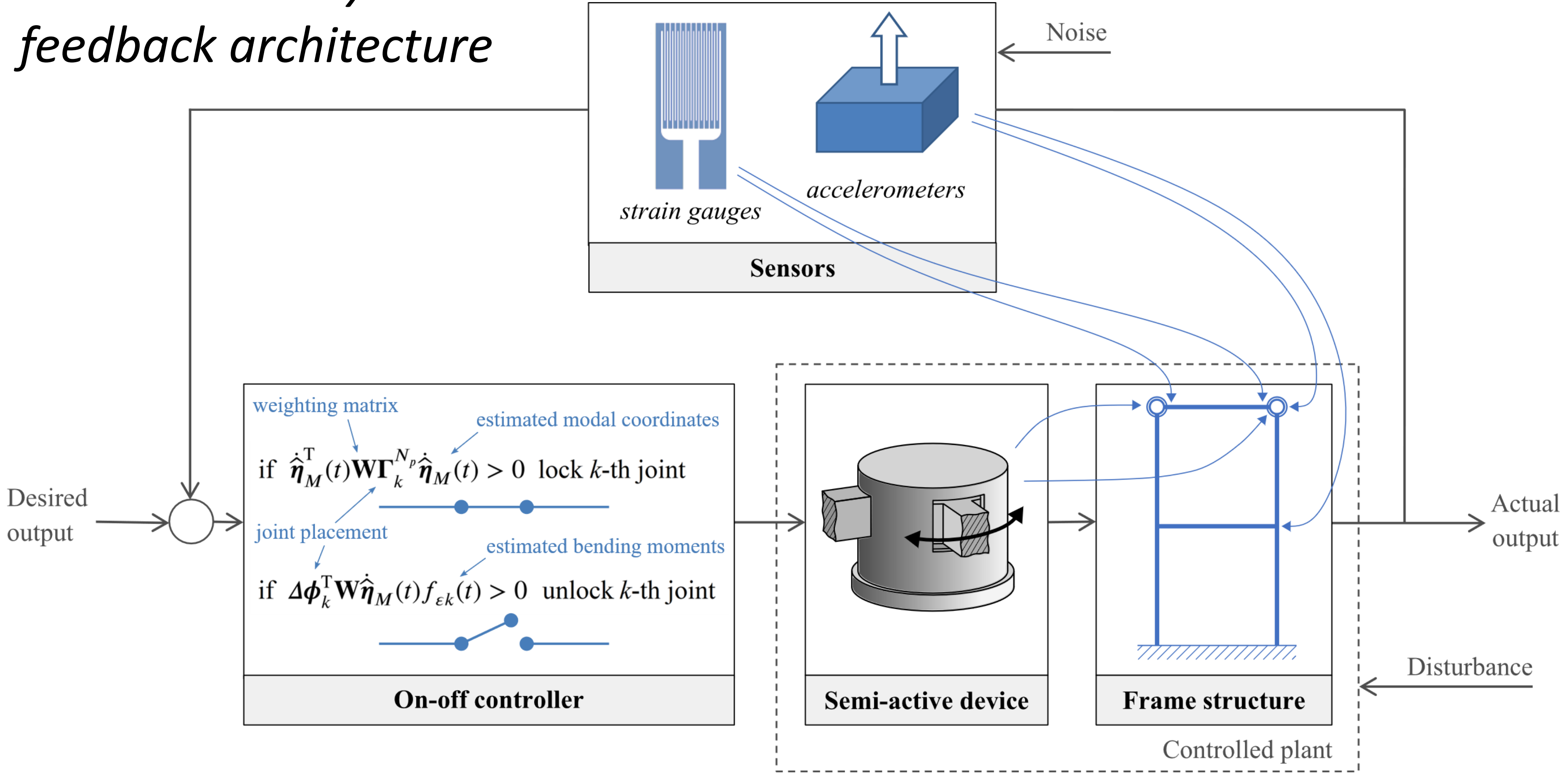
At each time instant t_i and for each kth lockable joint the optimization problem below is solved

$$(\mathbb{P}v)\quad \begin{array}{ll} \text{find} & u_k(t_i),\qquad k=1,2,\dots,N_k \\ \text{to minimize} & \displaystyle\sum_{p=1}^{N_p}\alpha_p\dot{W}_{pk}(t,u_k) \\ \text{subject to} & u_k(t_i)\in\{0,1\} \end{array}$$

2.3.4. Flowchart



2.3.4. Control system with feedback architecture



2.3.5. Structure with multiple lockable joints

$$\dot{W}_m = \dot{\eta}_m(t) \sum_{k=1}^{N_k} \Delta\phi_k f_k(t, \boldsymbol{\eta}, \dot{\boldsymbol{\eta}}, u_k)$$

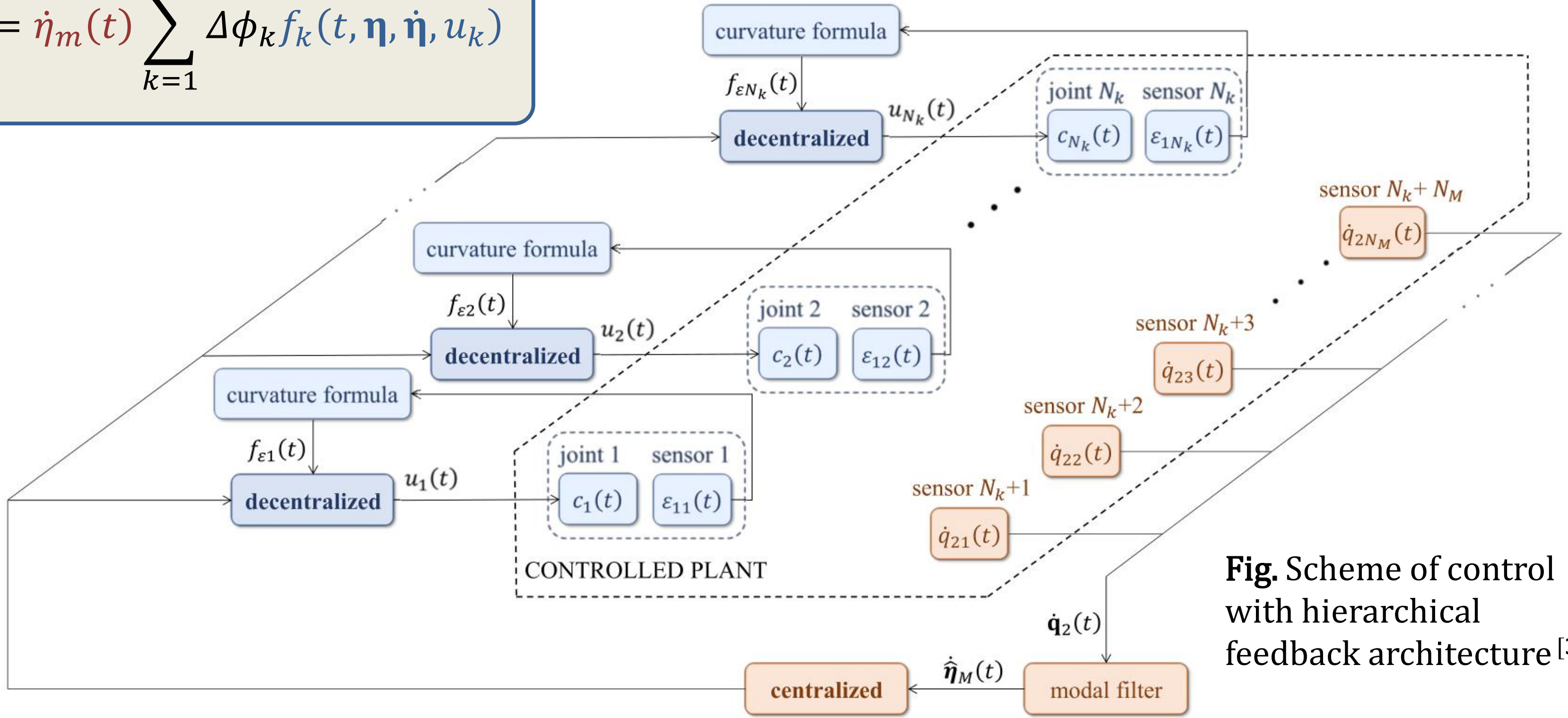


Fig. Scheme of control with hierarchical feedback architecture [3]

[3] Ostrowski M. et al. **Structural Control and Health Monitoring**. 2021; 28:e2710.

2.3.6. Potential applications

Developed control strategy has potential in two types of applications:

1. Vibration mitigation by shifting the energy into the higher-order vibration modes, usually characterised by higher material damping.

2. Enhancement of the energy harvesting process by transfer the total vibration energy to the preselected vibration mode optimal for the energy harvester

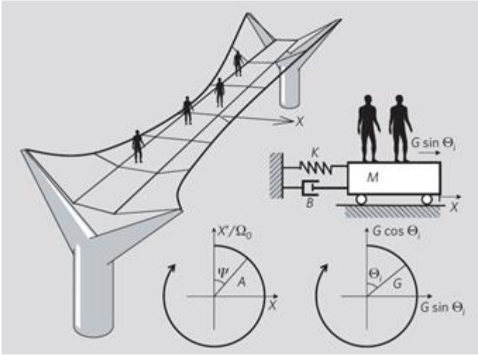


Fig. Vibration mitigation system in Millenium Bridge, London [1]

[1] Strogatz, S., Abrams, D., McRobie, A. et al. **Nature**. 2005; 438:43–44

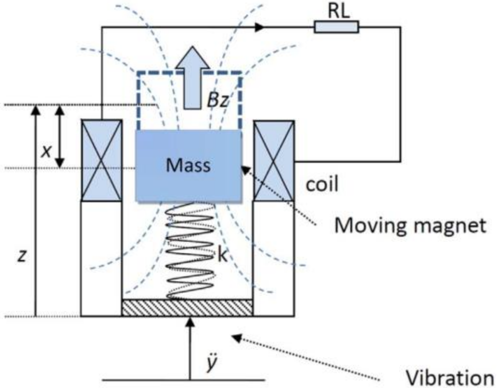


Fig. Example of electromagnetic energy harvester [2]

[2] Wei Ch. and Jing X. **Renewable and Sustainable Energy Reviews.** 2017; 74:1-18.

2.3.6a Vibration attenuation

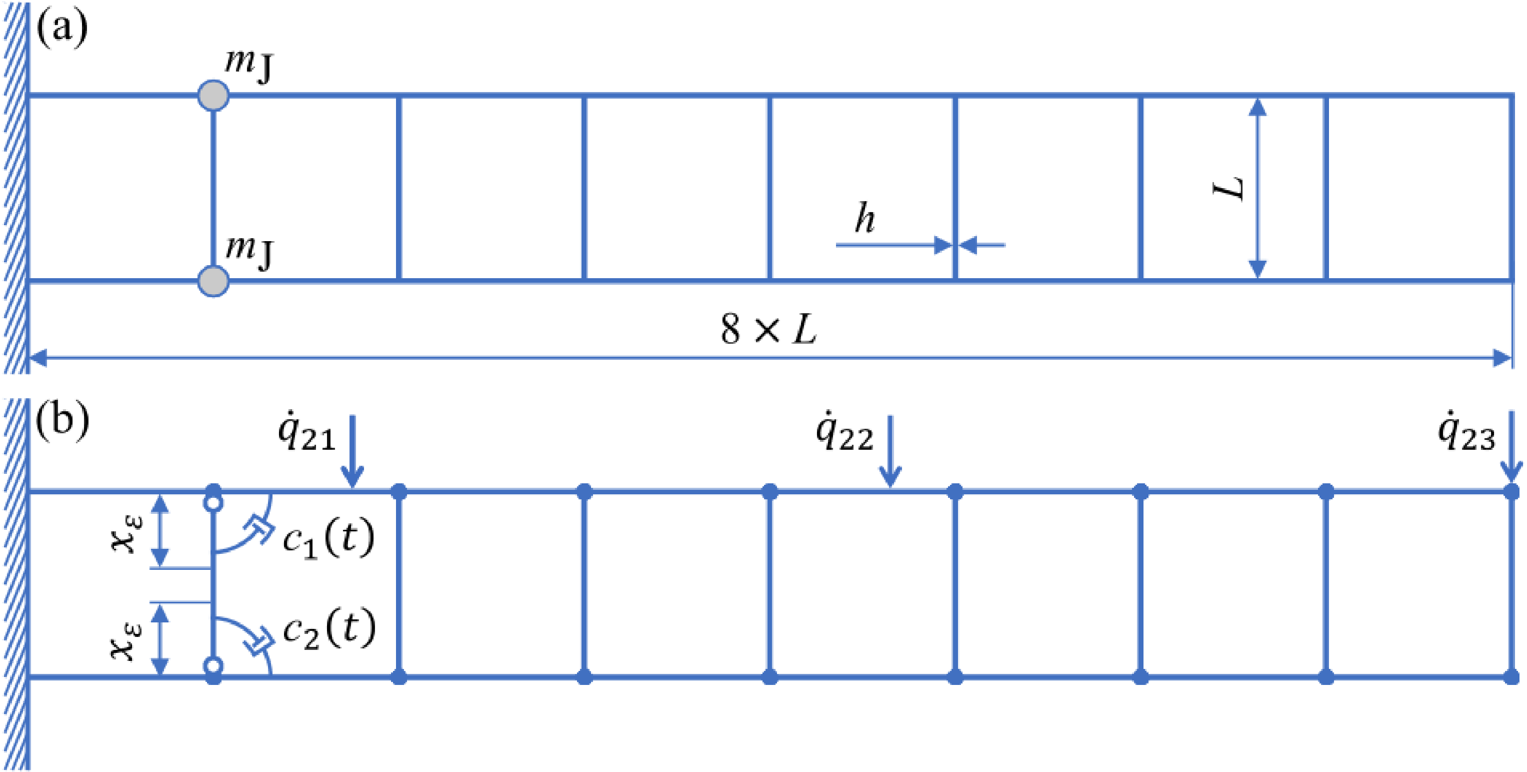


Fig. (a) scheme of the flexible frame structure (b) finite element model and sensor locations

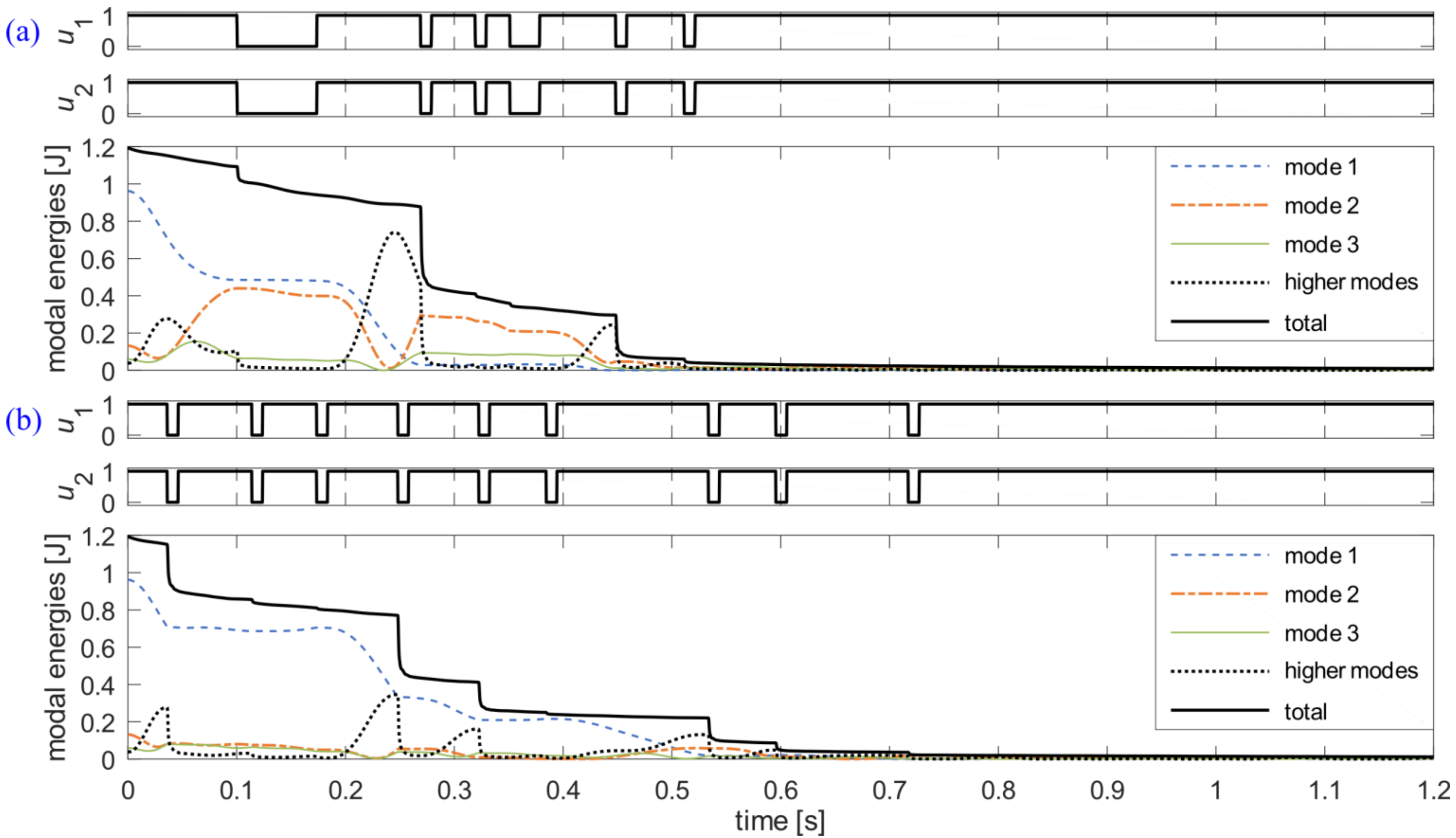


Fig. Comparison of time histories of control signals and modal energies obtained by: (a) proposed semi-active modal approach and (b) PAR control

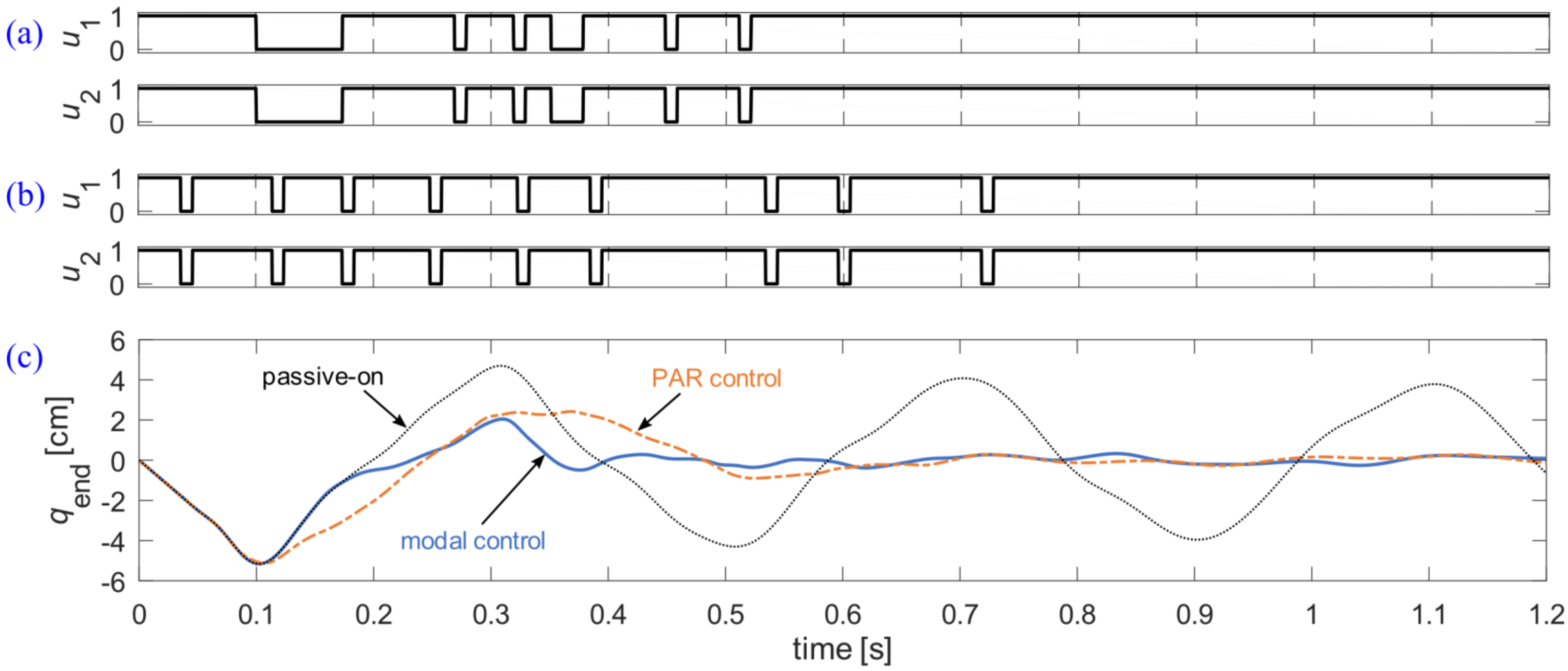


Fig. (a) time history of control signals obtained by the proposed approach, (b) by PAR and (c) comparison of control signals and displacement of the structure tip obtained by proposed semi-active modal approach, PAR control and fully locked (”passive-on”) case

2.3.6b Energy harvesting from seismically excited structure

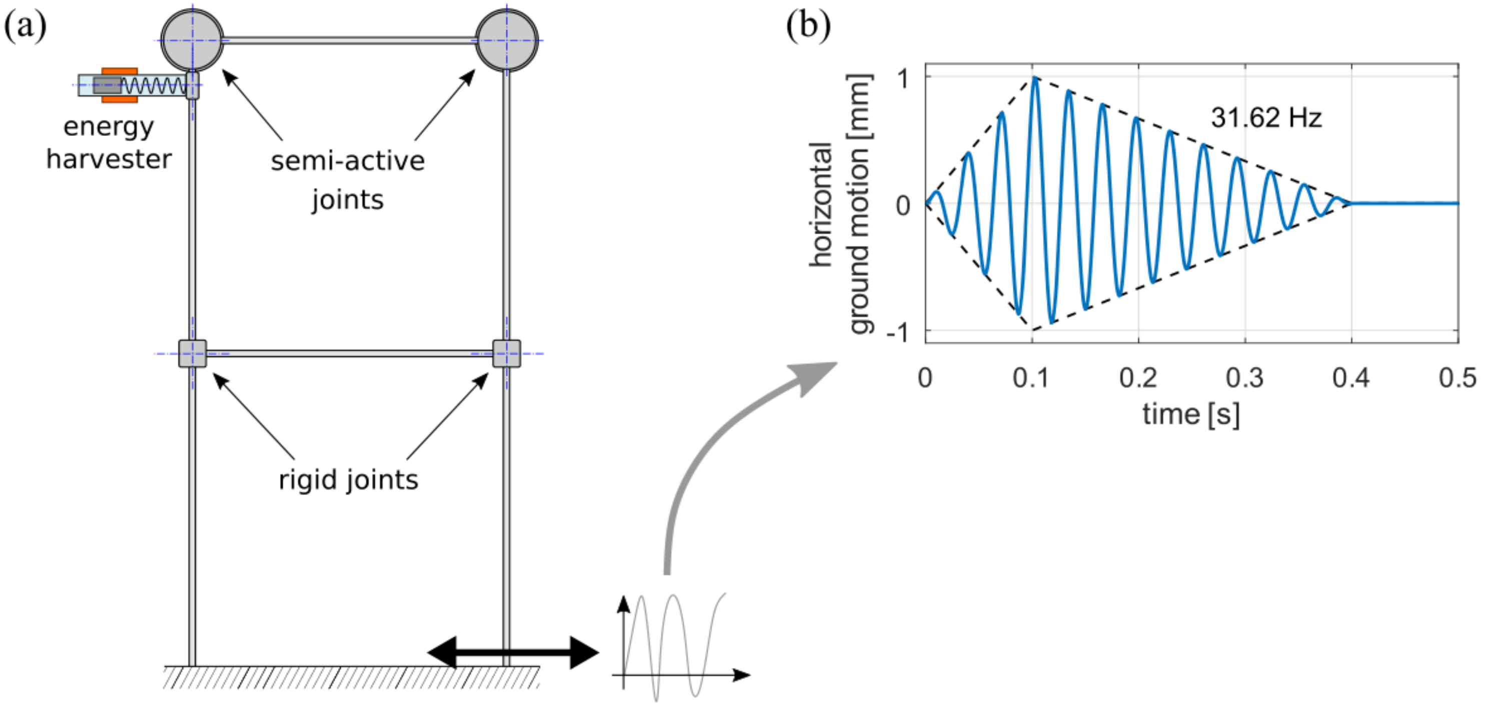


Fig. (a) kinematic structure excitation, (b) time history of the excitation

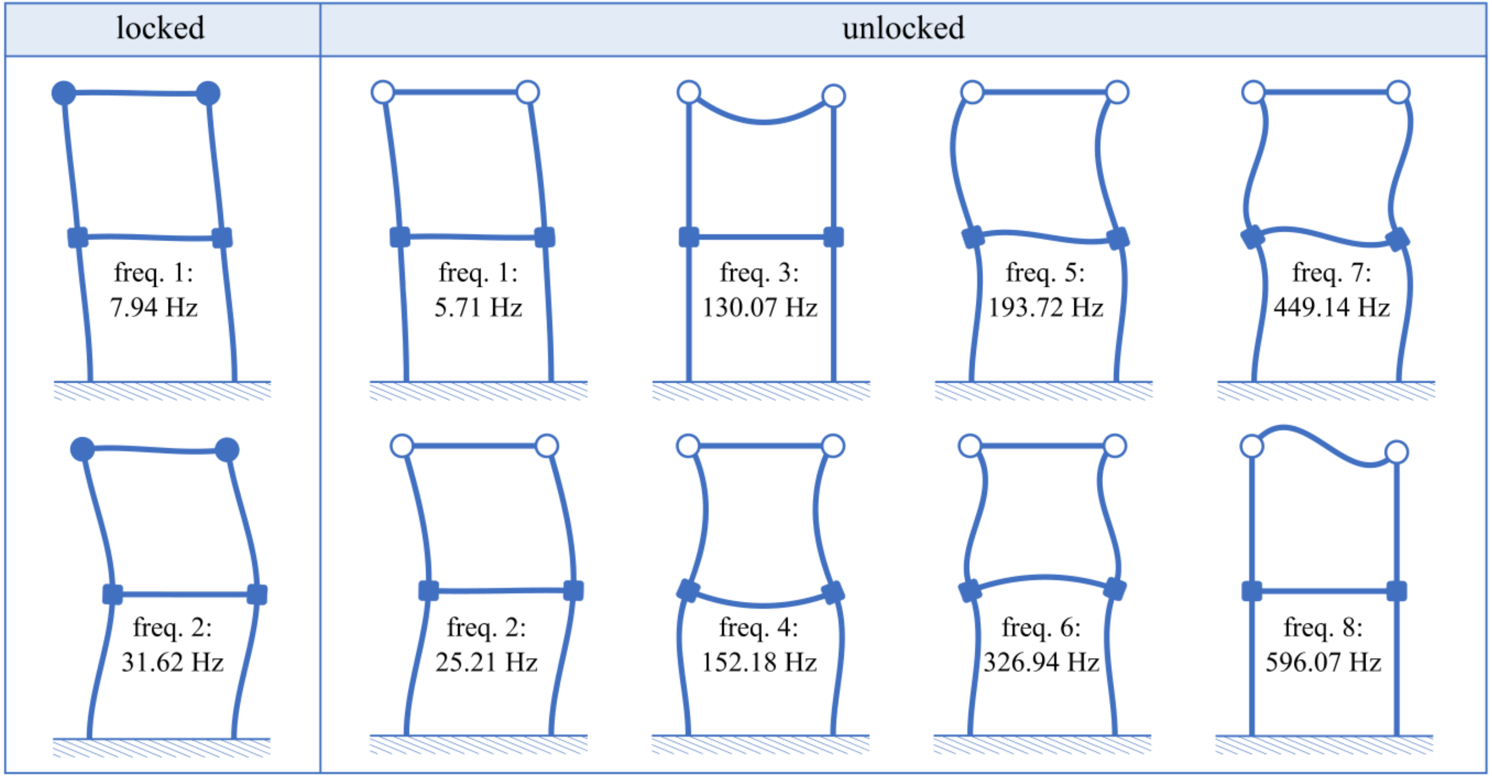


Fig. Mode shapes of the controlled structure without the energy harvester

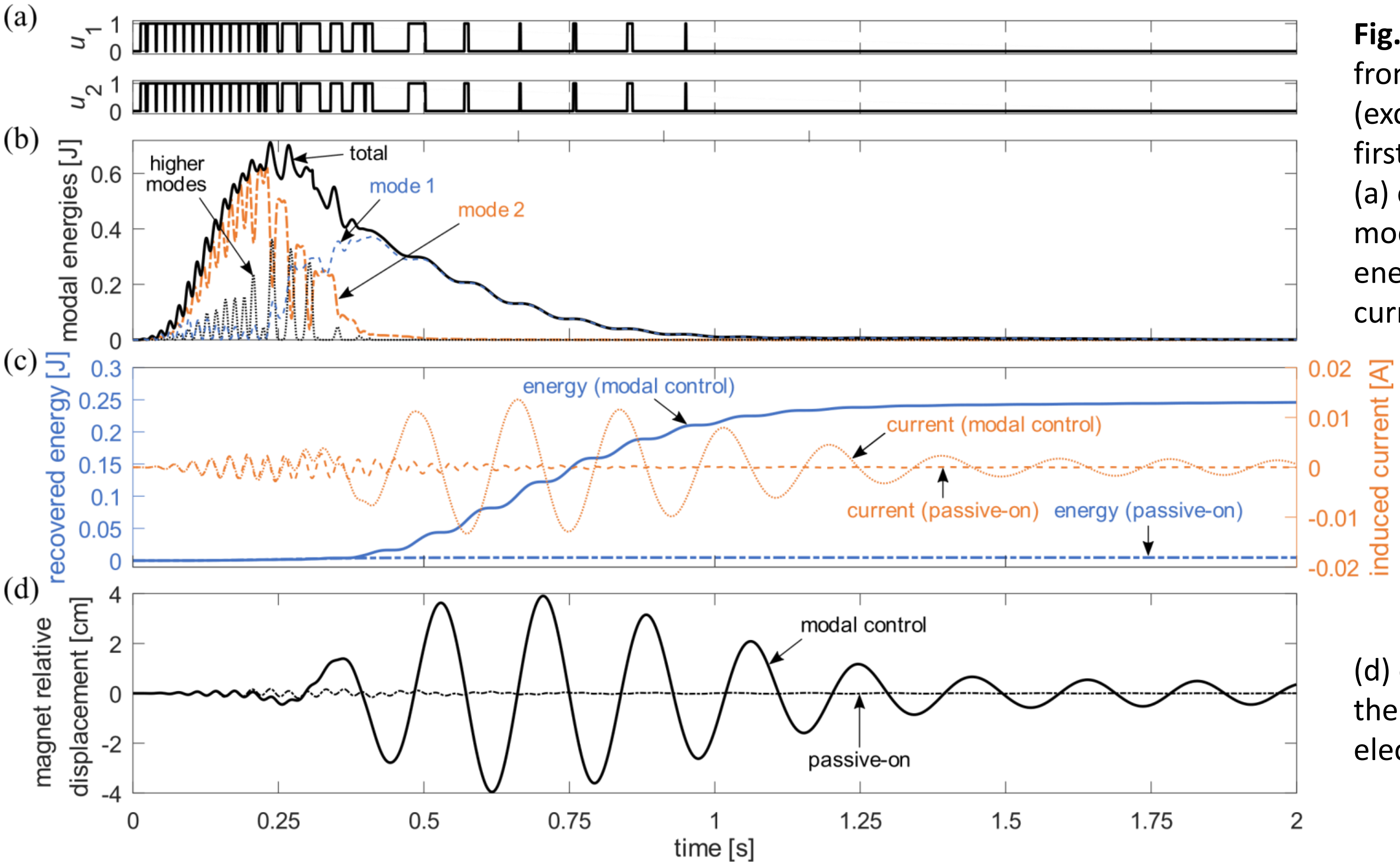


Fig. Energy transfer from the second (excited) mode to the first one:
(a) control signals, (b) modal energies, (c) energy and electrical current

(d) displacement of the magnet inside the electromagnetic coil

2.3.7. Conclusions for part III

Proposed semi-active modal control allows for:

- directed energy transfer between vibration modes
- vibration attenuation by transfer of the energy into high-order modes
- design the structure as the adaptive (controlled) energy buffer for energy harvester

3. Conclusions

Part I) Structural topology optimization

- A new optimal design method was presented in the field of elastoplasticity
- In proposed method stresses are introduced directly into optimization process

Part II) Optimal sensor placement

- An efficient method for deployment of dense sensor network over large structures was presented
- A sensor density function allowed to find the optimal solution in an iterative way instead of sequential removal of individual DOFs

Part III) Semi-actively lockable joints

- Proposed control strategy allows for the energy transfer between vibration modes
- It attenuates vibration by transfer of its energy into high-order modes

Dziękuję za uwagę!