Adaptacyjno-predykcyjne algorytmy sterowania układów semi-aktywnych poddanych nieznanym obciążeniom udarowym

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Presentation outline:

I. **Adaptive Impact Absorption (AIA)**

II. **Controllable dampers for AIA**

- operating principle, mathematical model

III. **Classical vs. state-dependent formulation**

IV. **Investigation of state-dependent formulation**

- application of Pontryagin's principle
- application of direct methods

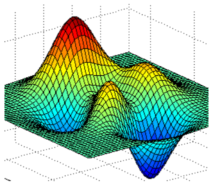
V. **Adaptive and predictive control algorithms**

- construction of predictive model
- time-continuous approach
- control function and response parametrisation

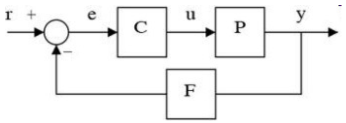
VI. **Conclusions**

I. Adaptive Impact Absorption (AIA)

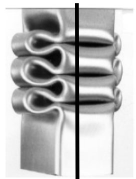
Basic motivation: development of novel, efficient and robust systems for **dissipation of impact energy**



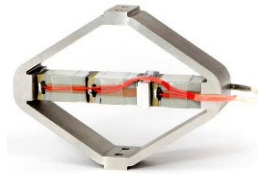
Optimization methods and control theory



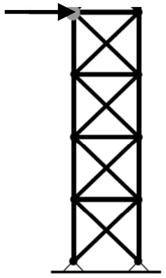
Impact Mechanics



Functional materials and control electronics



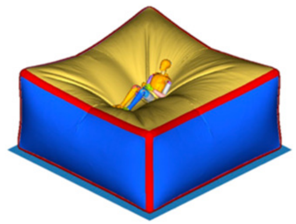
Adaptive Impact Absorption (AIA)
design and control of various energy-absorbing structures which can be adapted in real-time to actual dynamic loading



Adaptive skeletal structures

Adaptive dampers

Adaptive inflatable structures



Yield stress control in barriers, bumpers, railway cars

Hydraulic based on functional fluids or controllable valves

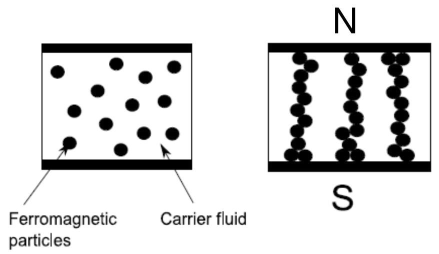
Pneumatic with controllable valves

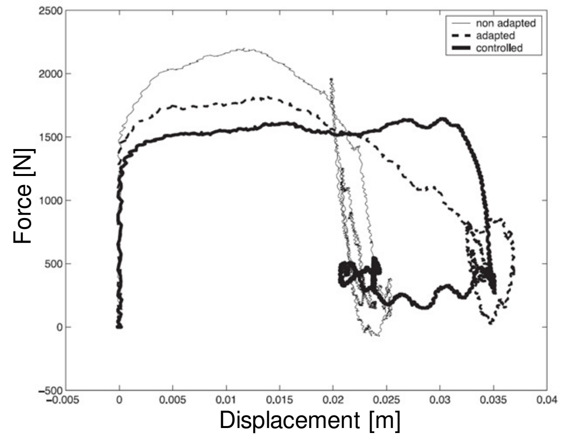
Adaptive airbags, rescue cushions

I. Adaptive Impact Absorption (AIA)

Design of adaptive MR dampers:

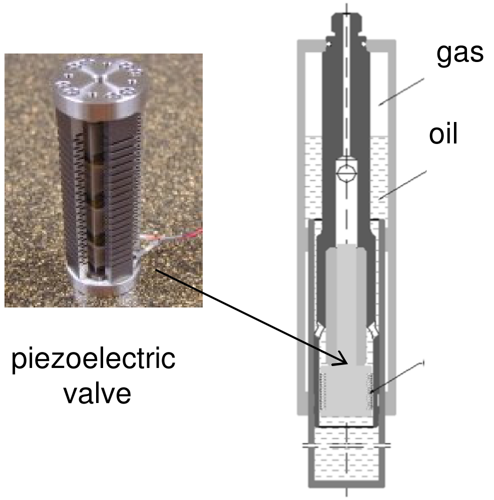
Adaptive absorbers based on MR-fluids,
G.Mikułowski, 2007

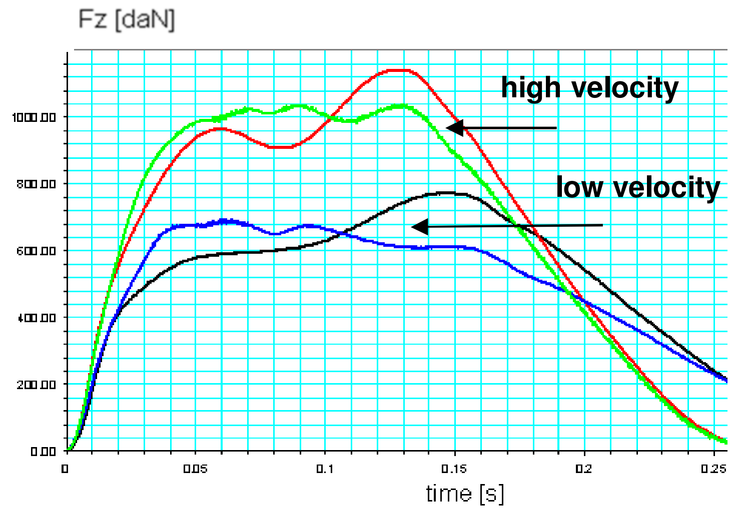




Design of adaptive hydro-pneumatic dampers:

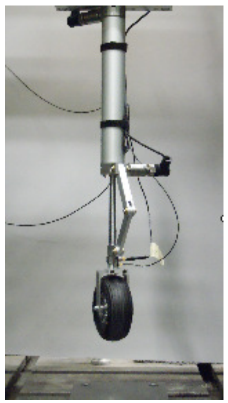
1. Adaptive landing gears, **EU Project ADLAND, 2003-2007**
2. ALG*:* optimum control strategy, G.Mikułowski, Ł.Jankowski, 2009

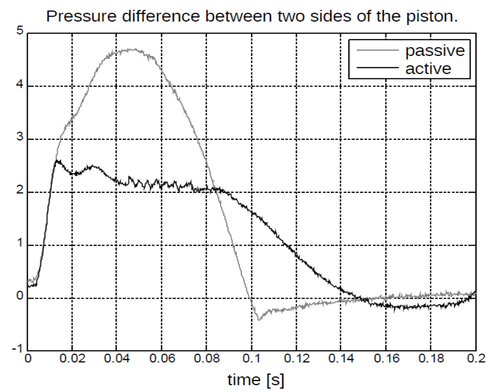




Design of adaptive pneumatic dampers:

1. Mathematical models, optimization, C.Graczykowski, 2012
2. Piezoelelectric valve design and characterisation, R.Wiszowaty, G.Mikułowski, 2013
3. Damper design and experimental testing, **R.Wiszowaty, 2016**





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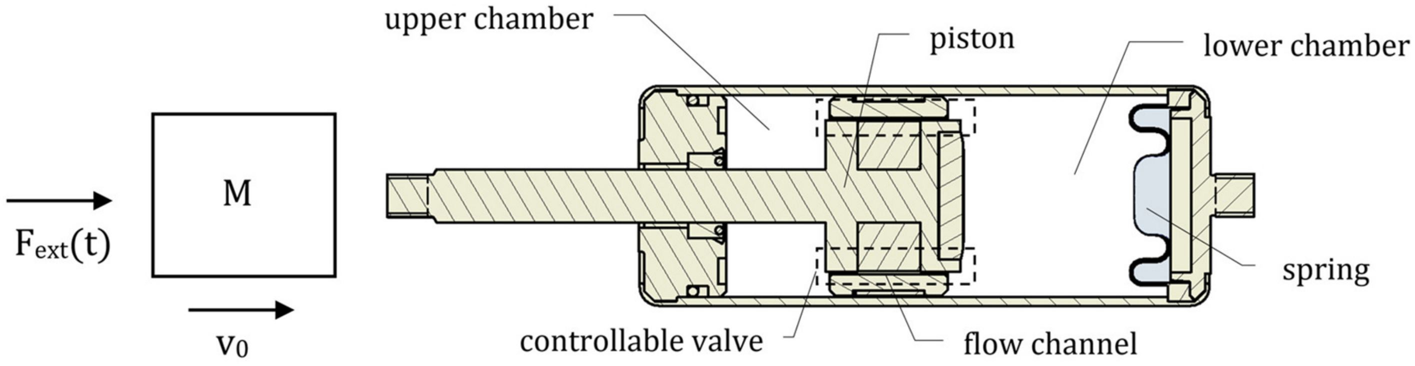
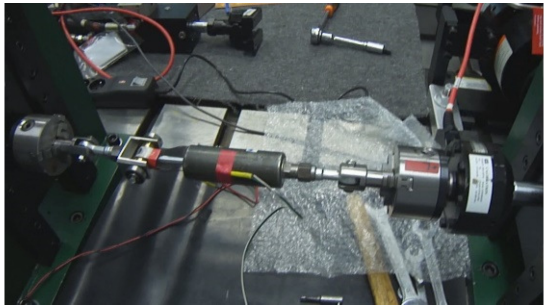
VI. **Conclusions**

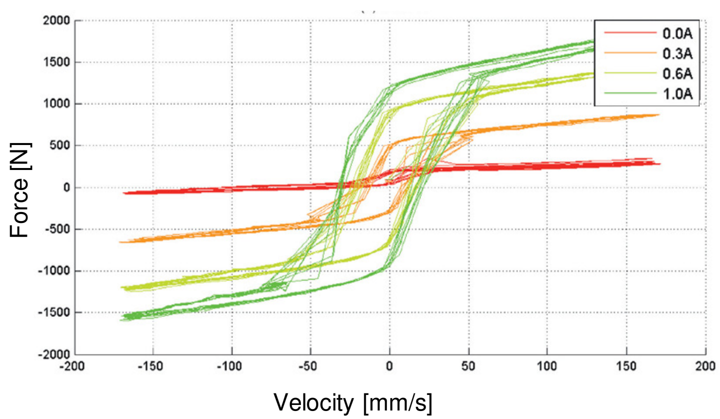
II. Controllable dampers for AIA

A. Operating principle and characteristics

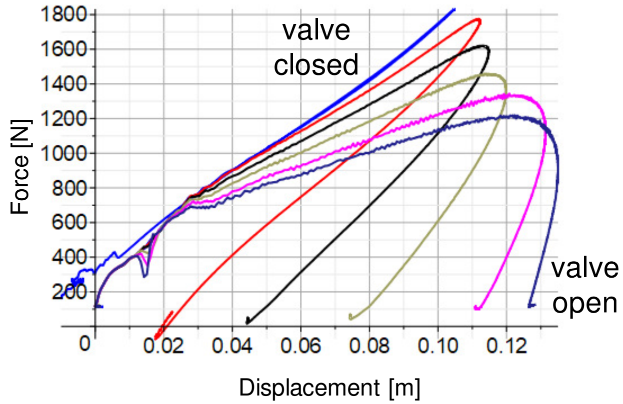


MR damper



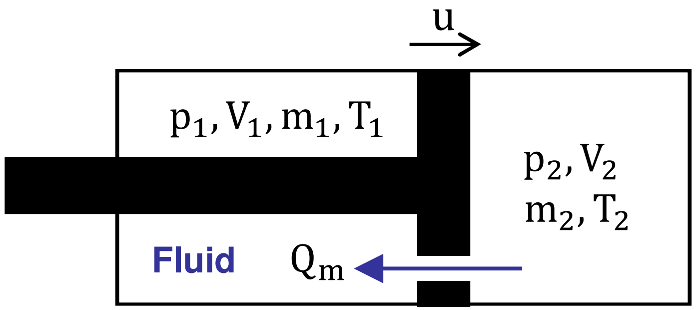


Pneumatic damper
with piezoelectric valve



II. Controllable dampers for AIA

B. Mathematical model of controllable damper



Damper model: $F(u(t), A_v(t))$

piston displacement

valve opening area

1. Model of valve flow: PDEs governing steady-state flow $\rightarrow$ $v_f(p, T, x, y)$

Space integration of $v_f(p, T, x, y)$ $\rightarrow$ $Q_V(p, T, A_v)$ or $Q_m(p, T, A_v)$

2. Model of thermodynamic processes in both chambers

Main assumptions:
- homogeneity of fluid parameters (p, T, ρ) in each chamber
- fluid compressibility: $\rho = \rho(p, T)$

Conservation laws:		Constitutive and geometric laws:
• Balance of fluid volume or mass (2 ODEs) • Therm. balance of fluid energy (2 ODEs)	+	• Equations of state (2 AEs) • Volumes definitions (2 AEs)

3. Definition of generated reaction force: $\boxed{F = p_2 A_2 - p_1 A_1}$

II. Controllable dampers for AIA

B. Mathematical model of controllable damper

- Balance of fluid volume: $V = V(p, T, m)$ $\rightarrow$ $\boxed{\dot{V} = \frac{\partial V}{\partial p}\dot{p} + \frac{\partial V}{\partial T}\dot{T} + \frac{\partial V}{\partial m}\dot{m}}$

$\beta = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_{T,m}, \quad \alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{p,m}$ $\rightarrow$ $\dot{V} + \beta V\dot{p} - \alpha V\dot{T} + Q_V = 0$

- Thermodynamic energy balance: $\boxed{\delta Q + dm\overline{H} = d(m\overline{U}) + \delta W}$

$$\delta Q = \gamma A(T_{ext} - T), \quad \overline{H} = c_p T_v + (1 - \alpha T_v)\frac{V}{m}p_v, \quad d\overline{U} = \left(c_p - \alpha p\frac{V}{m}\right)dT + (\beta p - \alpha T)\frac{V}{m}dp, \quad \delta W = p\delta V$$

- System of governing equations:

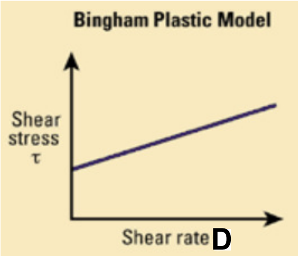
$$\begin{cases} \dot{V}_1 + \beta_1 V_1 \dot{p}_1 - \alpha_1 V_1 \dot{T}_1 + Q_V^1 = 0 & \Leftrightarrow \quad \dot{m}_1 = -Q_m \\ \dot{V}_2 + \beta_2 V_2 \dot{p}_2 - \alpha_2 V_2 \dot{T}_2 + Q_V^2 = 0 & \quad \dot{m}_2 = Q_m \\ \dot{Q} + \dot{m}c_p T_v + Q_V p_v(1 - \alpha T_v) = \dot{m}c_p T - \alpha p T Q_V + Q_V p(\beta p - \alpha T) + m c_p \dot{T} - \alpha p V \dot{T} + V\dot{p}(\beta p - \alpha T) + p\dot{V} & \text{- inflow chamber} \\ \dot{Q} + Q_V p = Q_V p(\beta p - \alpha T) + m c_p \dot{T} - \alpha p V \dot{T} + V\dot{p}(\beta p - \alpha T) + p\dot{V} & \text{- outflow chamber} \end{cases}$$

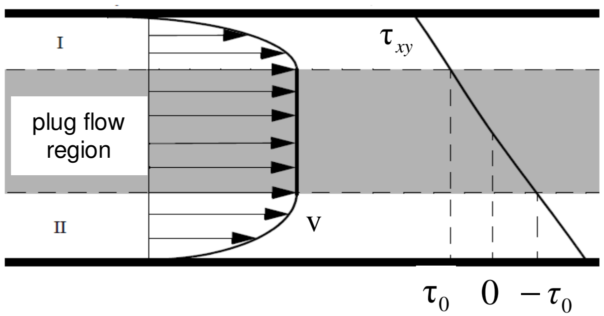
Final model: **4 ODEs** in terms of variables chosen from: $p_1, p_2, T_1, T_2, m_1, m_2$

II. Controllable dampers for AIA

C. Specification for MR damper

- Valve flow model: 2D viscous incompressible flow



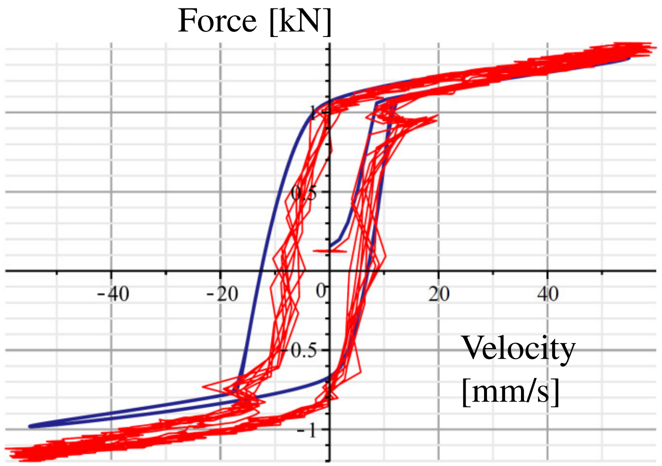


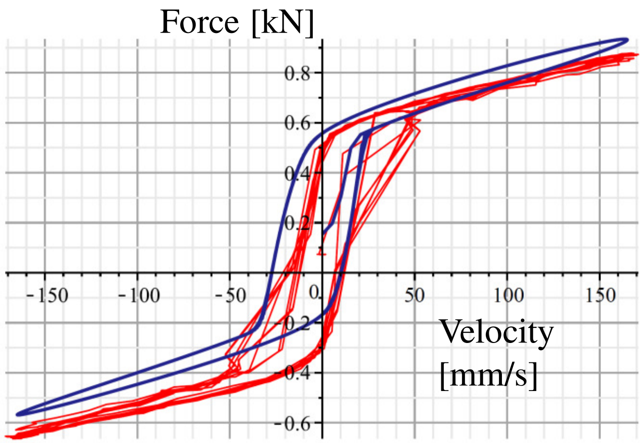
$\Delta p \geq \Delta p^{crit}$: viscous flow + plug flow
$\Delta p < \Delta p^{crit}$: blocking of the flow

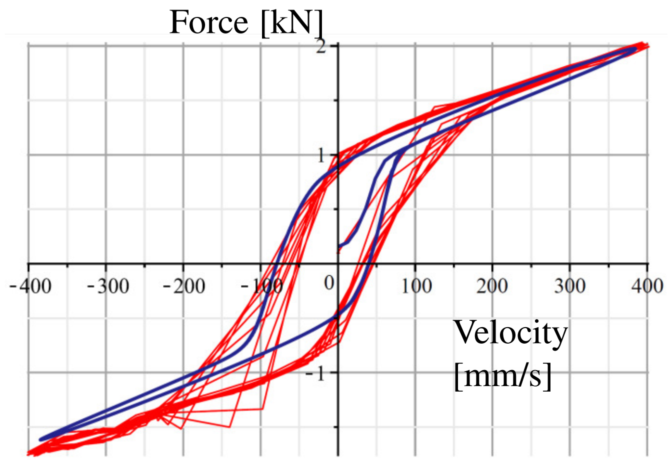
$$Q_V(\Delta p) \quad \text{for} \quad \Delta p \geq \Delta p^{crit}$$
$$Q_V = 0 \quad \text{for} \quad \Delta p < \Delta p^{crit}$$

- Constitutive model: MR fluid decomposed to:
  - classical viscous fluid
  - compressible fluid

2 ODEs - balance of fluid **volume**
2 ODEs - balance of fluid **energy**







Graczykowski C., Pawłowski P., Exact physical model of magnetorheological damper, Applied Mathematical Modelling, DOI: 10.1016/j.apm.2017.02.035, Vol.47, pp.400-424, 2017

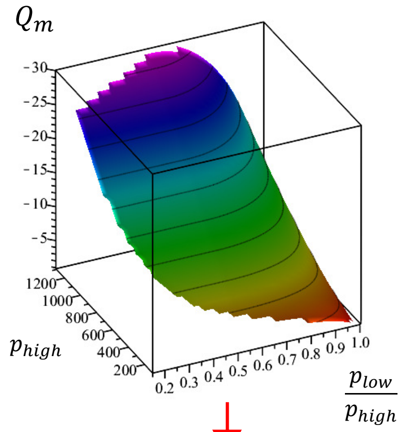
II. Controllable dampers for AIA

D. Specification for pneumatic damper

- Valve flow model: 1D compressible inviscid flow

$p_{low}/p_{high} \geq k_{crit}$: Saint-Venant flow model: $Q_m(p_{high}, T_{high}, p_{low}, A_v)$

$p_{low}/p_{high} < k_{crit}$: choked flow model: $Q_m(p_{high}, T_{high}, A_v)$



- Constitutive model:

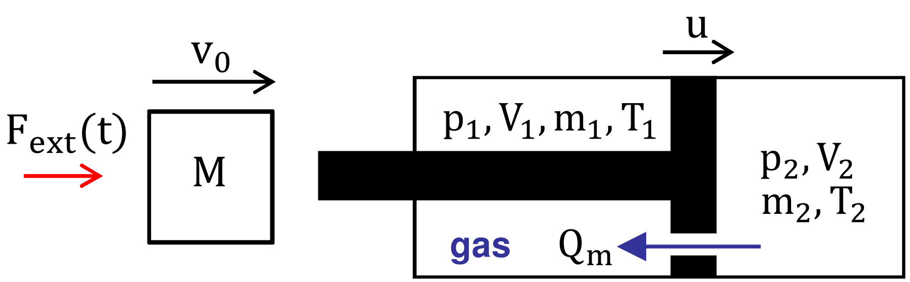
$pV - mRT = 0$

Compressibility: $\beta = 1/p$

Thermal expansion: $\alpha = 1/T$

2 ODEs - balance of fluid **mass**
2 ODEs - balance of fluid **energy**

- Complete model of the impact problem:



$$M\ddot{u} + (p_2 A_2 - p_1 A_1) + F(u, v) = F_{ext}$$
$$\dot{m}_1 = Q_m(p_1, p_2, T_2)$$
$$\dot{m}_2 = -Q_m(p_1, p_2, T_2)$$
$$\dot{Q}_1 + \dot{m}_1 c_p T_2 = \dot{m}_1 c_v T_1 + m_1 c_v \dot{T}_1 + p_1 \dot{V}_1$$
$$\dot{Q}_2 + \dot{m}_2 c_p T_2 = \dot{m}_2 c_v T_2 + m_2 c_v \dot{T}_2 + p_2 \dot{V}_2$$
$$p_1 V_1 = m_1 R T_1, \ p_2 V_2 = m_2 R T_2$$
$$V_1 = A_1(h_1^0 + u), V_2 = A_2(h_2^0 - u)$$
$$\text{IC}: u(0) = u_0, \dot{u}(0) = v_0,$$
$$p_1(0) = p_1^0, \ p_2(0) = p_2^0, \ T_1(0) = T_1^0, \ T_2(0) = T_2^0$$

II. Controllable dampers for AIA

D. Specification for pneumatic damper

- State-space models:

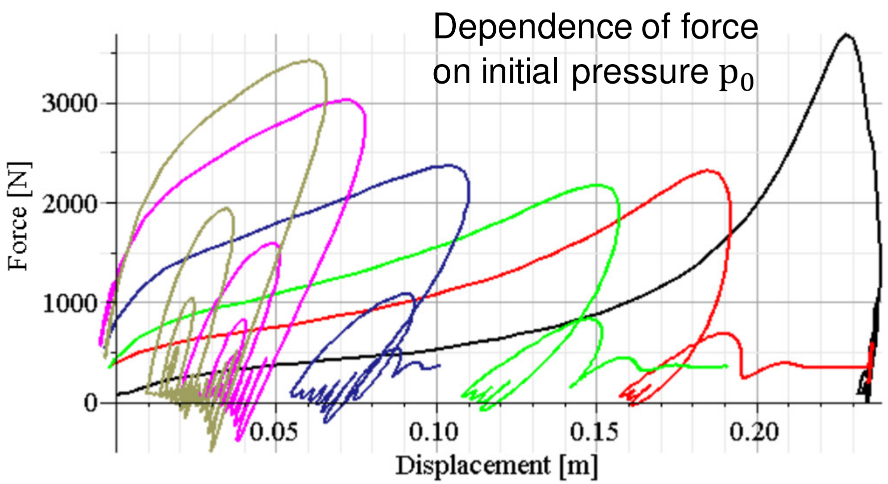
Variables: u, v, p_1, p_2, T_1, T_2

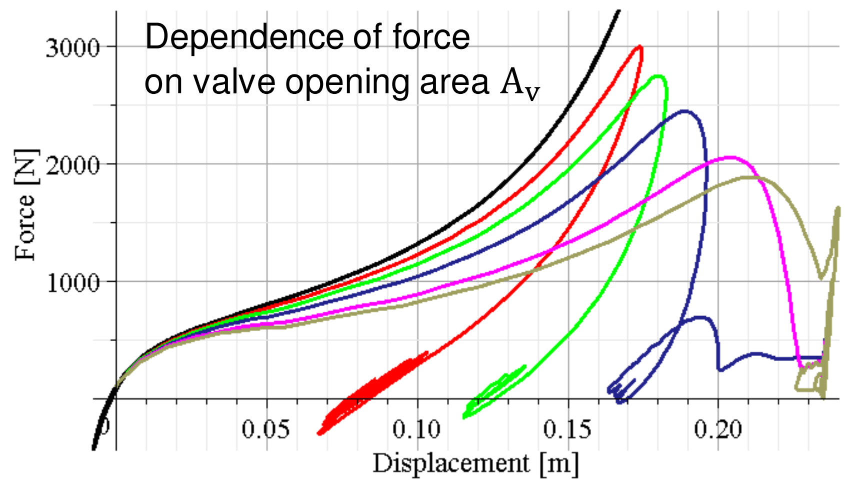
$$\frac{du}{dt} = v$$
$$\frac{dv}{dt} = M^{-1}\left[F_{ext}(t) - F_p(p_1, p_2) - F(u, v)\right]$$
$$\frac{dp_1}{dt} = \frac{\kappa}{V_1}\left[-p_1\dot{V}_1(v) + Q_m R T_2\right]$$
$$\frac{dp_2}{dt} = \frac{\kappa}{V_2}\left[-p_2\dot{V}_2(v) - Q_m R T_2\right]$$
$$\frac{dT_1}{dt} = \frac{RT_1}{c_v p_1 V_1}\left[Q_m(c_p T_2 - c_v T_1) - p_1\dot{V}_1(v)\right]$$
$$\frac{dT_2}{dt} = -\frac{RT_2}{c_v p_2 V_2}\left[Q_m R T_2 + p_2\dot{V}_2(v)\right]$$

Variables: u, v, m_1, m_2, T_1, T_2

$$\frac{du}{dt} = v$$
$$\frac{dv}{dt} = M^{-1}[\ldots]$$
$$\frac{dm_1}{dt} = Q_m(m_1, T_1, T_2)$$
$$\frac{dm_2}{dt} = -Q_m(m_1, T_1, T_2)$$
$$\frac{dT_1}{dt} = \frac{RT_1}{c_v p_1 V_1}\left[Q_m(c_p T_2 - c_v T_1) - \frac{m_1 R T_1}{V_1}\dot{V}_1(v)\right]$$
$$\frac{dT_2}{dt} = -\frac{RT_2}{c_v p_2 V_2}\left[Q_m R T_2 + \frac{(m-m_2)RT_2}{V_2}\dot{V}_2(v)\right]$$

- Selected results:





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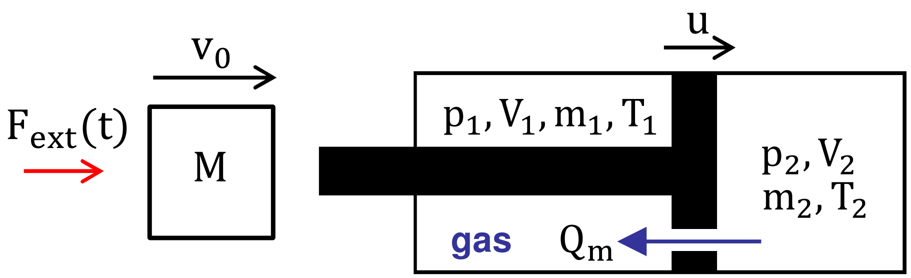
III. Classical vs. state-dependent formulation

A. Classical variational formulation of AIA problem

- Considered **adaptive system**:



Assumptions:

- impact excitation is known or identified
- no system disturbances (forces or leakages)
- no constraints on maximal valve opening

- Straightforward formulation: $\boxed{\text{Minimize: } \max\left(F_{abs} - F_{ext}\right)}$

- Classical formulation:

$$\text{Minimize:} \quad \int_0^T \left(F_{abs}\left(A_v(t)\right) - F_{abs}^{opt}(t)\right)^2 dt$$

optimal change of absorber's force

$$\text{With respect to:} \quad A_v(t) \geq 0$$

$$\text{Subject to:} \quad \int_{u_0}^{u(T)} F_{abs} du = E_{imp}^0 + E_{imp}^{ext} = \frac{1}{2} M v_0^2 + \int_{u_0}^{u(T)} F_{ext} du$$

- Specification to impact of rigid object ($F_{ext} = 0$): $u(T) = d, \quad F_{abs}^{opt} = \frac{M v_0^2}{2d}$

$$\text{Minimize:} \quad \int_0^T \left(F_{abs}\left(A_v(t)\right) - \frac{M v_0^2}{2d}\right)^2 dt$$

III. Classical vs. state-dependent formulation

B. Two-step solution of the classical AIA problem

1. The problem of finding feasible path

Minimize: $$\int_0^{u(T)} \left(F_{abs}^{feas}(u) - F_{abs}^{opt}\right)^2 du$$

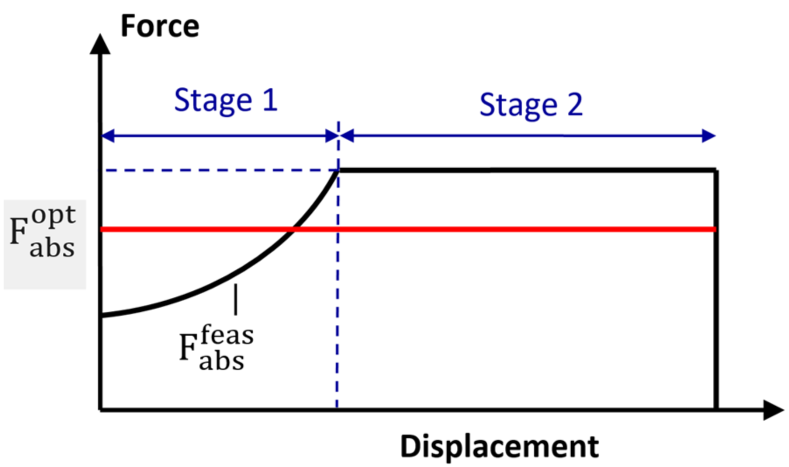
With respect to: $F_{abs}^{feas}(u) \geq 0$

Subject to: $$\int_{u_0}^{u(T)} F_{abs}^{feas}(u) du = E_{imp}^0$$

- Three-stage solution

$1.\, A_v = 0 \rightarrow$ max. increase of $F_{abs}^{feas}(u)$,

$2.\, F_{abs}^{feas}(u) = \text{const.}$, $3.\, F_{abs}^{feas}(u) \rightarrow 0$



2. The problem of path-tracking

Minimize: $$\int_0^{T} \left(F_{abs}(A_v) - F_{abs}^{feas}\right)^2 dt$$

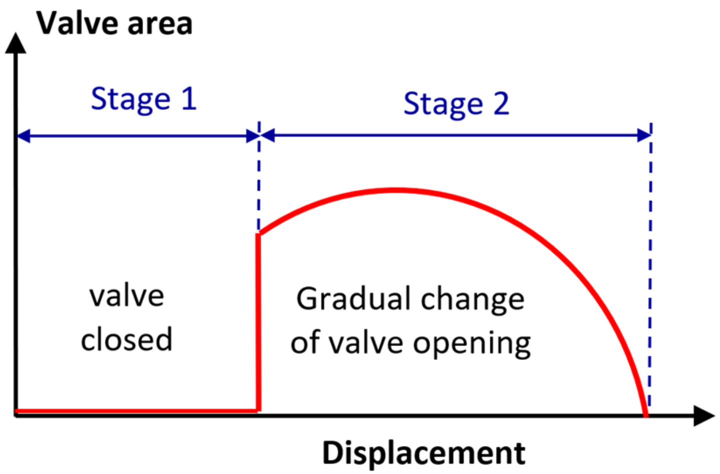
- Solution using inverse dynamics

Ideal path-tracking: $F_{abs}(A_v) = F_{abs}^{feas}$

+ system model:

→ $A_v(t)$ - open-loop control

$A_v(u)$ - semi-passive system

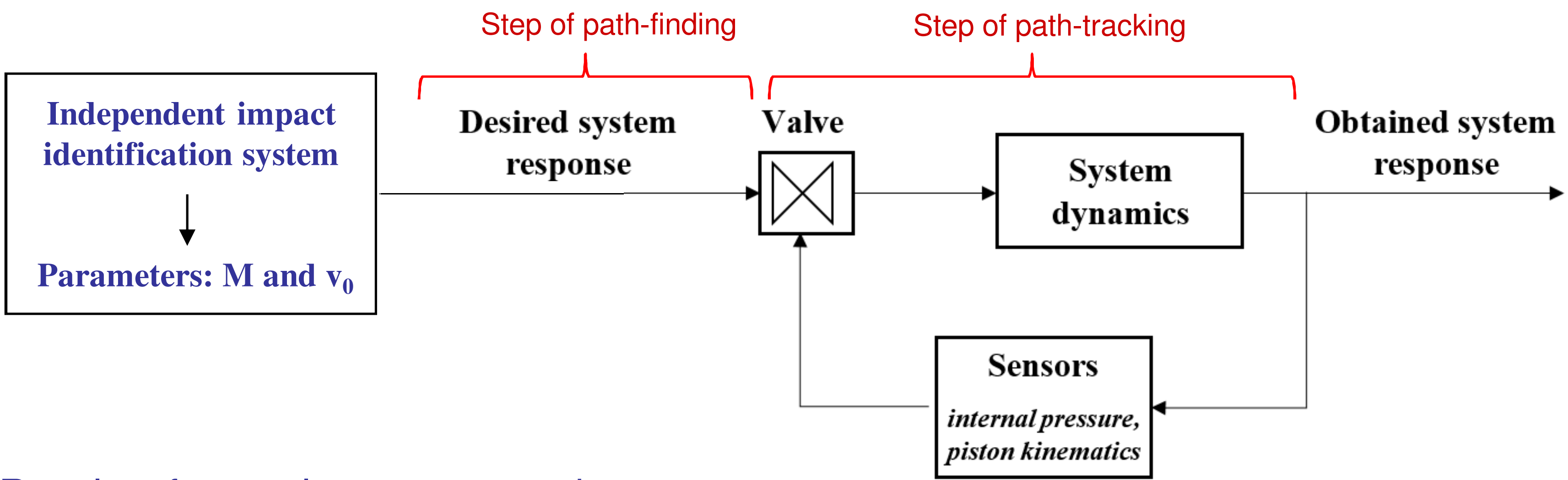


- Solution based on feedback control

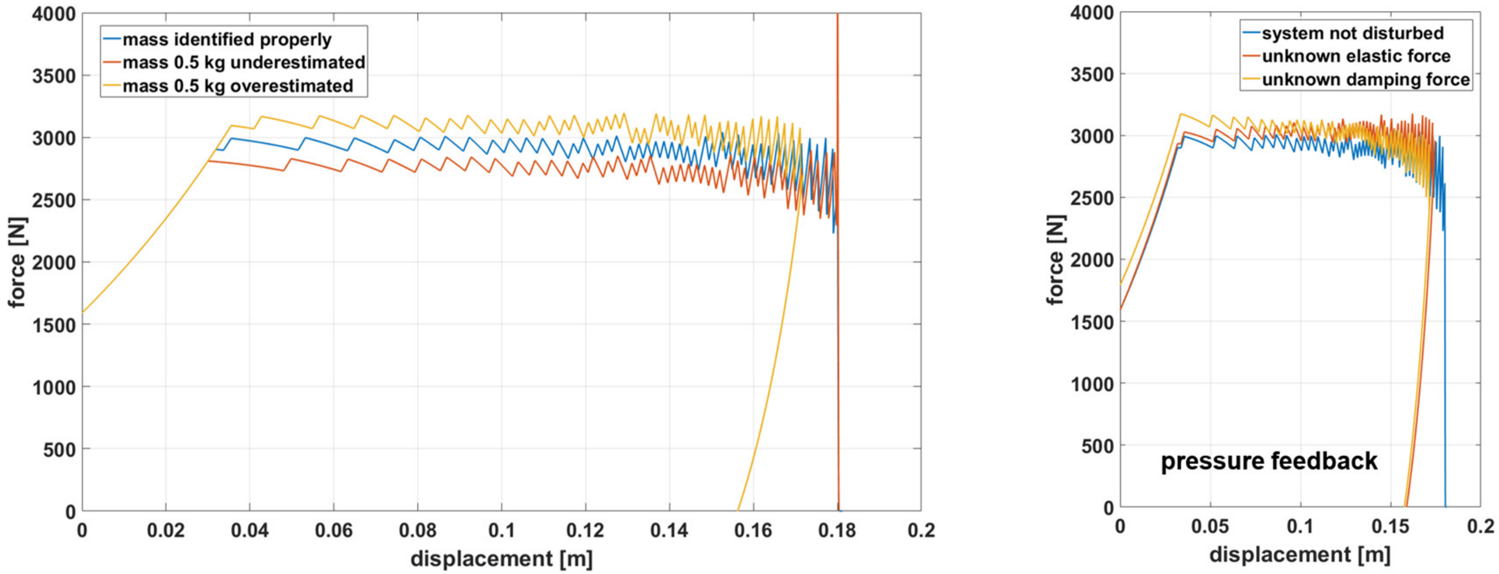
III. Classical vs. state-dependent formulation

C. Standard control systems for AIA and their drawbacks

- Scheme of closed-loop control system:



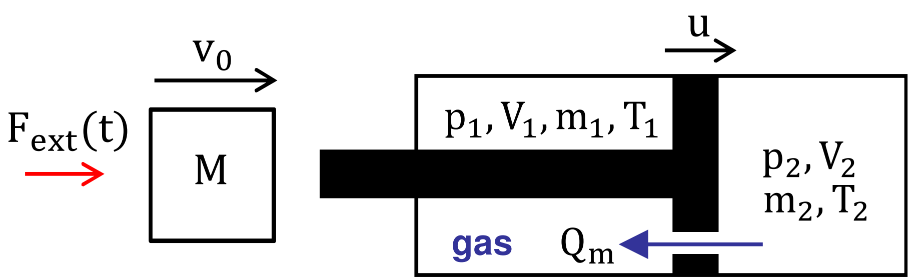
- Results of control system operation:



III. Classical vs. state-dependent formulation

D. State-dependent formulation of AIA problem

- Considered **self-adaptive system**:



Requirements:

- automatic adaptation to unknown excitation
- robustness to process disturbances
- accounting for constraints of valve opening

- Basic concept: **actual optimal value of absorber's force determined during impact**
- State-dependent formulation: Minimize: $\int_0^T \left(F_{abs}(A_v(t), t) - F_{abs}^{opt}(u, v, t)\right)^2 dt$

With respect to: $0 \leq A_v(t) \leq A_v^{max}$

Subject to: $\int_{u_0}^{u(T)} F_{abs} du = E_{imp}^0 + E_{imp}^{ext} = \frac{1}{2} M v_0^2 + \int_{u_0}^{u(T)} F_{ext} du$

Absorber's force: $F_{abs} = F_p(A_v(t)) + F_{dist}(t)$

Optimal force: $F_{abs}^{opt} = \frac{M\dot{u}(t)^2}{2(d-u(t))} + F_{ext}(t)$

- Force-based state-dependent path-tracking:

$$\text{Minimize:} \quad \int_0^T \left(F_p\left(A_v(t)\right) - \frac{M\dot{u}(t)^2}{2\left(d - u(t)\right)} - [F_{ext}(t) - F_{dist}(t)]\right)^2 dt$$

III. Classical vs. state-dependent formulation

E. Discretisation of state-dependent formulation

- Application of Model Predictive Control (MPC)

Series of problems: $$\text{Minimize: } \int_{t_i}^{T} \left(F_p\big(A_\nu(t)\big) - \frac{M\dot{u}(t_i)^2}{2\big(d - u(t_i)\big)} - [F_{ext}(t_i) - F_{dist}(t_i)] \right)^2 dt$$

$$\text{With respect to: } \quad 0 \le A_\nu(t) \le A_\nu^{max}, \; t \in (t_i, T)$$

$$\text{Subject to: } \quad \int_{u(t_i)}^{u(T)} F_{abs} du = \tfrac{1}{2} M\dot{u}(t_i)^2 + \int_{u(t_i)}^{u(T)} F_{ext} du$$

- Shortening of prediction interval:

$$\text{Minimize: } \int_{t_i}^{t_i+\Delta t} \left(F_p\big(A_\nu(t)\big) - \frac{M\dot{u}(t_i)^2}{2\big(d - u(t_i)\big)} - [F_{ext}(t_i) - F_{dist}(t_i)] \right)^2 dt$$

refers to single control step

refers to entire absorption process

- Application of equation of motion: $M\ddot{u}(t) + F_p(t) = F_{ext}(t) - F_{dist}(t)$

- Kinematics-based state-dependent path-tracking:

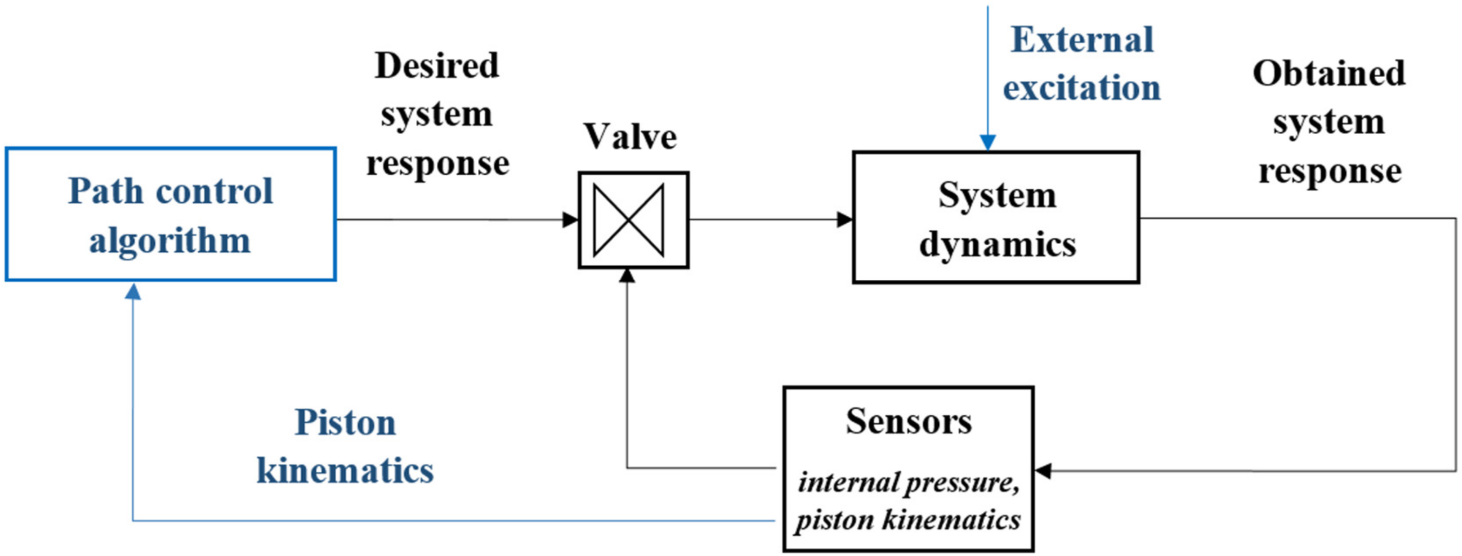
$$\text{Minimize: } \int_{0}^{T} \left(\ddot{u}(t) + \frac{\dot{u}(t)^2}{2\big(d - u(t)\big)} \right)^2 dt \;\rightarrow\; \text{Minimize: } \int_{t_i}^{t_i+\Delta t} \left(\ddot{u}(t) + \frac{\dot{u}(t_i)^2}{2\big(d - u(t_i)\big)} \right)^2 dt$$

III. Classical vs. state-dependent formulation

F. Control systems for self-adaptive AIA and their advantages

- Scheme of closed-loop control system:

The presence of state-dependent term in the objective function implies presence of **additional closed loop** in control system



- Advantages: - lack of impact identification system, - robustness to various disturbances

Graczykowski C., Faraj R., Development of control systems for fluid-based adaptive impact absorbers, Mechanical Systems and Signal Processing, DOI: 10.1016/j.ymssp.2018.12.006, Vol.122, pp.622-641, 2019

Faraj R., Graczykowski C., Hybrid Prediction Control for self-adaptive fluid-based shock-absorbers, Journal of Sound and Vibration, DOI: 10.1016/j.jsv.2019.02.022, Vol.449, pp.427-446, 2019

Presentation outline:

I. **Adaptive Impact Absorption (AIA)**

II. **Controllable dampers for AIA**

- operating principle, mathematical model

III. **Classical vs. state-dependent formulation**

IV. **Investigation of state-dependent formulation**

- application of Pontryagin's principle
- application of direct methods

V. **Adaptive and predictive control algorithms**

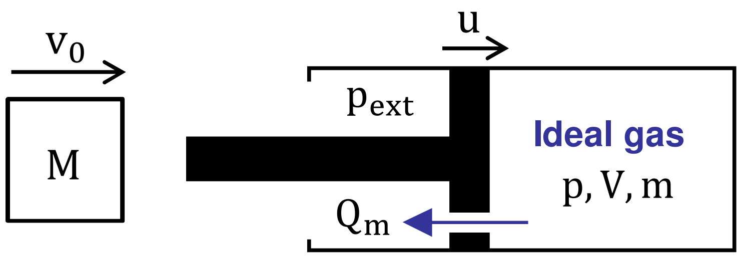
- construction of predictive model
- time-continuous approach
- control function and response parametrisation

VI. **Conclusions**

IV. Investigation of state-dependent formulation

A. Considered model and formulation

- Considered system and its basic model



- Single-chamber damper
- Isothermal process
- Simple model of the gas flow

$$M\ddot{u} + (p - p_{ext})A = 0$$
$$\dot{m} = -A_v(p - p_{ext})$$
$$pV = mRT,\ V = A(d-u)$$
$$\text{IC: } u(0) = u_0, \dot{u}(0) = v_0, m(0) = m_0$$

$\longrightarrow$

$$\frac{du}{dt} = v$$
$$\frac{dv}{dt} = -\frac{mRT}{M(d-u)} + \frac{p_{ext}A}{M}$$
$$\frac{dm}{dt} = -A_v\left(\frac{mRT}{A(d-u)} - p_{ext}\right)$$
$$\text{IC: } u(0) = u_0, \dot{u}(0) = v_0, m(0) = m_0$$

- State-dependent path-tracking problem

$$\text{Minimize: } \int_0^T \left(\ddot{u} + \frac{\dot{u}^2}{2(d-u)}\right)^2 dt \quad \longrightarrow \quad \int_0^T \left(\frac{mRT}{M(d-u)} - \frac{p_{ext}A}{M} - \frac{v^2}{2(d-u)}\right)^2 dt$$

$$\text{With respect to: } \quad m(t) \in \langle m^{min}, m^{max} \rangle$$
$$A_v(t) \in \left\langle {A_v}^{min}, {A_v}^{max} \right\rangle$$

$$\text{Subject to: } \quad \int_{u_0}^{u(T)} F_{abs} du = \tfrac{1}{2} M v_0^2$$

IV. Investigation of state-dependent formulation

B. Recapitulation of Pontryagin's maximum principle

Problem considered:

Maximize: $J(x, \bar{u}) = \int_{t_0}^{t_f} g(x, \bar{u}, t)dt$

With respect to: $\bar{u}$

Subject to: $\dot{x} = f(x, \bar{u}, t)$

Hamiltonian: $H(x, \lambda, \bar{u}, t) = g(x, \bar{u}, t) + \lambda^T(t)\, f(x, \bar{u}, t)$

<u>Optimality conditions</u> for the case of free time t_f:

1. $H(x^*(t), \lambda^*(t), \bar{u}^*(t), t) \geq H(x^*(t), \lambda^*(t), \bar{u}(t), t)$
2. $\frac{\partial H}{\partial x}(x^*(t), \lambda^*(t), \bar{u}^*(t), t) = -\dot{\lambda}^*$
3. $\frac{\partial H}{\partial \lambda}(x^*(t), \lambda^*(t), \bar{u}^*(t), t) = \dot{x}^*$
4. Transversality conditions:
   $\lambda^*(t_f) = 0$ if $x(t_f)$ free or $x^*(t_f) = x(t_f)$ if $x(t_f)$ specified
5. $H(x^*(t_f), \lambda^*(t_f), \bar{u}^*(t_f), t_f) = 0$

IV. Investigation of state-dependent formulation

C. Application to simple AIA problem - formulation

Mass of gas as a control variable:

$$x = [\dot{u}, u]^T = [x_1, x_2]^T, \quad x_0 = [v_0, 0]^T, \quad \bar{u} = m(t)$$

Problem considered:

$$\text{Minimize: } J(x, \bar{u}) = \int_0^{t_f} \left[\left(\frac{\bar{u}RT}{M(d - x_2\,)} - \frac{p_{ext}A}{M} \right) - \frac{x_1^2}{2(d - x_2)} \right]^2 dt$$

$$\text{With respect to: } \bar{u} \in \langle 0, m_0 \rangle, \qquad \text{Subject to: } \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{\bar{u}RT}{M(d - x_2\,)} + \dfrac{p_{ext}A}{M} \\ x_1 \end{bmatrix}$$

$$\text{Hamiltonian: } H(x, \lambda, \bar{u}, t) = \left(-\frac{\bar{u}RT}{M(d-x_2\,)} + \frac{p_{ext}A}{M} \right) \lambda_1 + x_1\lambda_2 + \left[\left(\frac{\bar{u}RT}{M(d-x_2\,)} - \frac{p_{ext}A}{M} \right) - \frac{x_1^2}{2(d-x_2)} \right]^2$$

Optimality conditions:

1a. $H(x^*(t), \lambda^*(t), \bar{u}^*(t), t) \leq H(x^*(t), \lambda^*(t), \bar{u}(t), t)$

1b. Singular arc: $\dfrac{\partial H}{\partial \bar{u}} = 0 \quad \rightarrow \lambda_{1(sa)}^* = 2\left[\dfrac{\bar{u}_{(sa)}^* RT}{M(d-x_2^*)} - \dfrac{p_{ext}A}{M} - \dfrac{(x_1^*)^2}{2(d-x_2^*)} \right]$

$$\rightarrow \bar{u}_{(sa)}^* = \frac{M(x_1^*)^2}{2RT} + \frac{p_{ext}A(d-x_2^*)}{RT} + \frac{\lambda_{1(sa)}^* M(d-x_2^*)}{2RT}$$

IV. Investigation of state-dependent formulation

C. Application to simple AIA problem - governing equations

1. Control: $\bar{u}^* = \begin{cases} \bar{u}^{min} & \text{for } \lambda_1^* < 2\left[\frac{\bar{u}^{min}RT}{M(d-x_2^*)} - \frac{p_{ext}A}{M} - \frac{(x_1^*)^2}{2(d-x_2^*)}\right] \\ \bar{u}^{max} & \text{for } \lambda_1^* > 2\left[\frac{\bar{u}^{max}RT}{M(d-x_2^*)} - \frac{p_{ext}A}{M} - \frac{(x_1^*)^2}{2(d-x_2^*)}\right] \\ \bar{u}^*_{(sa)}(x_1^*, x_2^*, \lambda_1^*) & \text{for } \lambda_1^* \in \left[2\left(\frac{\bar{u}^{min}RT}{M(d-x_2^*)} - \cdots\right),\ 2\left(\frac{\bar{u}^{max}RT}{M(d-x_2^*)} - \cdots\right)\right] \end{cases}$

2. $$\dot{\lambda}_1^* = 2\left[\frac{\bar{u}^*RT}{M(d-x_2^*)} - \frac{p_{ext}A}{M} - \frac{(x_1^*)^2}{2(d-x_2^*)}\right]\frac{x_1^*}{d-x_2^*} - \lambda_2^*$$

$$\dot{\lambda}_2^* = -2\left[\frac{\bar{u}^*RT}{M(d-x_2^*)} - \frac{p_{ext}A}{M} - \frac{(x_1^*)^2}{2(d-x_2^*)}\right]\left(\frac{\bar{u}^*RT}{M(d-x_2^*)^2} - \frac{(x_1^*)^2}{2(d-x_2^*)^2}\right) + \frac{\bar{u}^*RT}{M(d-x_2^*)^2}\lambda_1^*$$

3. $$\dot{x}_1^* = -\frac{\bar{u}^*RT}{M(d-x_2^*)} + \frac{p_{ext}A}{M}$$

$$\dot{x}_2^* = x_1^*$$

4. Transversality conditions: $x_1^*(t_f) = 0,\ \lambda_2^*(t_f) = 0$

5. $$\left(-\frac{\bar{u}^*(t_f)RT}{M(d-x_2^*(t_f))} + \frac{p_{ext}A}{M}\right)\lambda_1(t_f) + \left(\frac{\bar{u}^*(t_f)RT}{M(d-x_2^*(t_f))} - \frac{p_{ext}A}{M} - \frac{x_1^*(t_f)^2}{2(d-x_2^*(t_f))}\right)^2 = 0$$

$+$ condition of energy disspation $\rightarrow \lambda_1^*(t_f) = 0$

Problem solved: Find $\{\lambda_1^*(0), \lambda_2^*(0)\}$ such that all above conditons are satisfied.

IV. Investigation of state-dependent formulation

C. Application to simple AIA problem - proposed solution method

Remarks:

1. Differentiation of the Hamiltonian with respect to time:

$$\frac{dH}{dt} = \frac{\partial H}{\partial \bar{u}}\frac{\partial \bar{u}}{\partial t} + \frac{\partial H}{\partial x}\dot{x} + \frac{\partial H}{\partial \lambda}\dot{\lambda}$$

$H(t) = \text{const.}$: i) at singular arc, ii) when $\bar{u}^* = u_{\min}$ or $\bar{u}^* = u_{\max}$.

2. Condition $H(t_f) = 0$, continuity of state, co−state and control:
   - $H(t) = 0$ for $t \in \langle 0, t_f \rangle$ $\rightarrow$ additional equation for costates,
   - reduction of the problem to finding $\lambda_1^*(0)$.

- The standard approach:

  Find $\lambda_1^*(0)$ using the transversality conds.: $\lambda_1^*(t_f) = 0,\ \lambda_2^*(t_f) = 0$

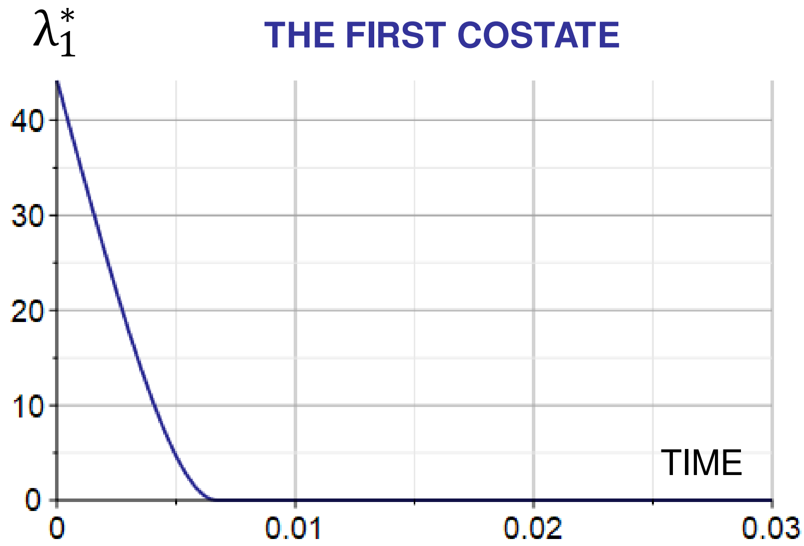
- The proposed approach:
  - assumption: the last stage follows optimal state-dependent path

    $\rightarrow$ control $\bar{u}^* = \frac{M(x_1^*)^2}{2RT} + \frac{p_{ext}A(d-x_2^*)}{RT}$ at singular arc with $\lambda_1^*(t) = \lambda_2^*(t) = 0$,
  - find $\lambda_1^*(0)$ using the conditions of reaching singular arc.

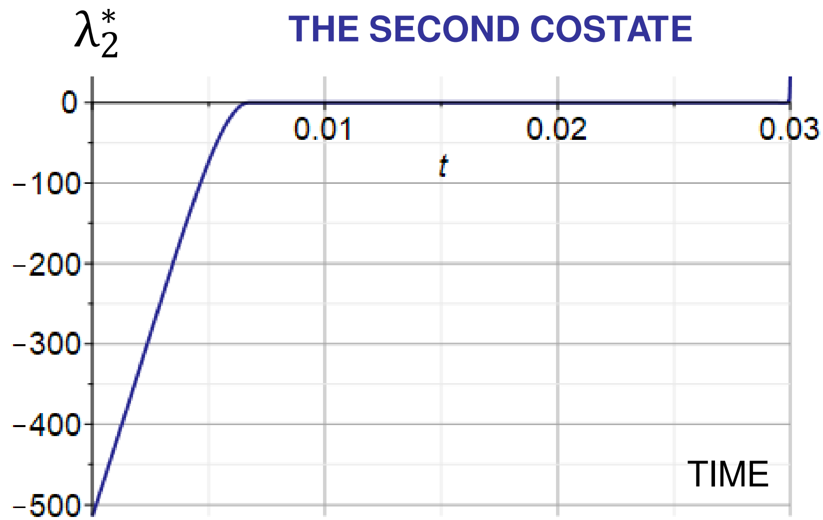
IV. Investigation of state-dependent formulation

C. Application to simple AIA problem - numerical results

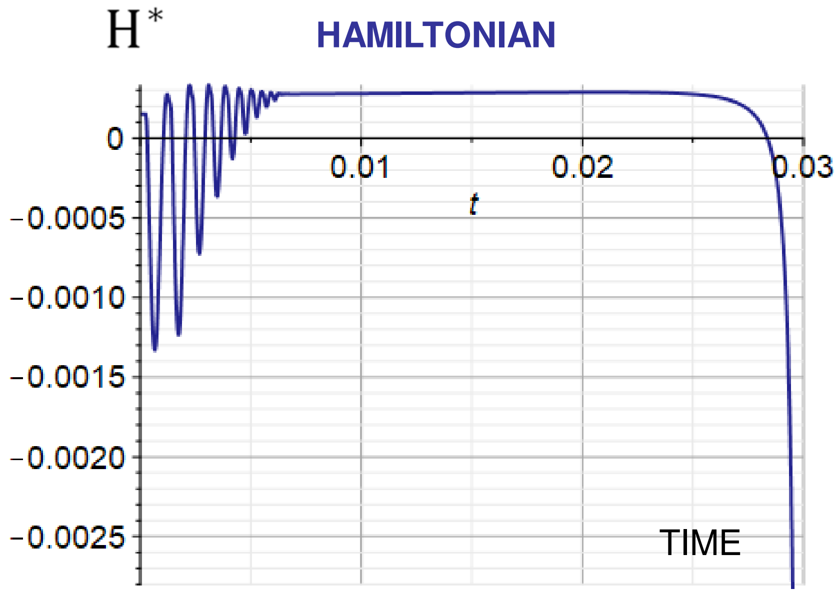
THE FIRST COSTATE



THE SECOND COSTATE

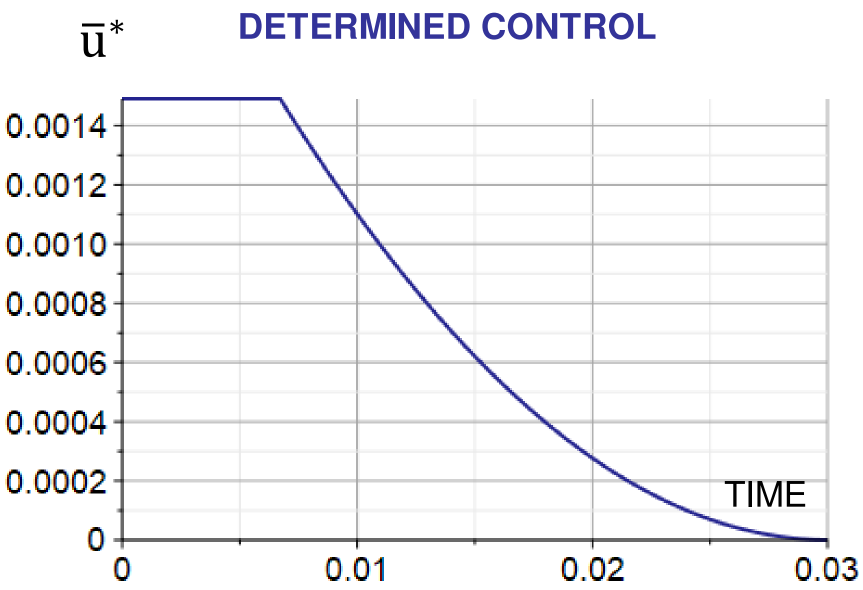


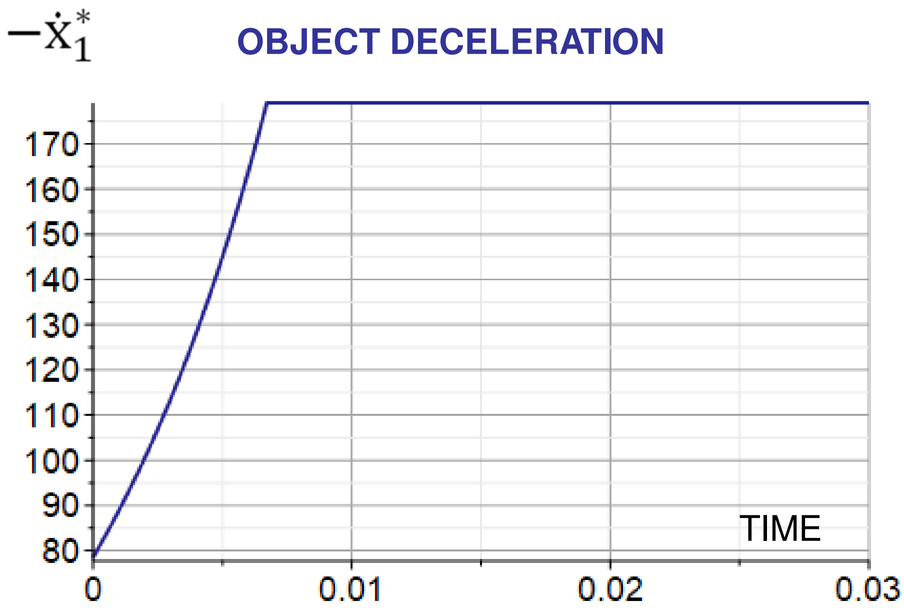
HAMILTONIAN

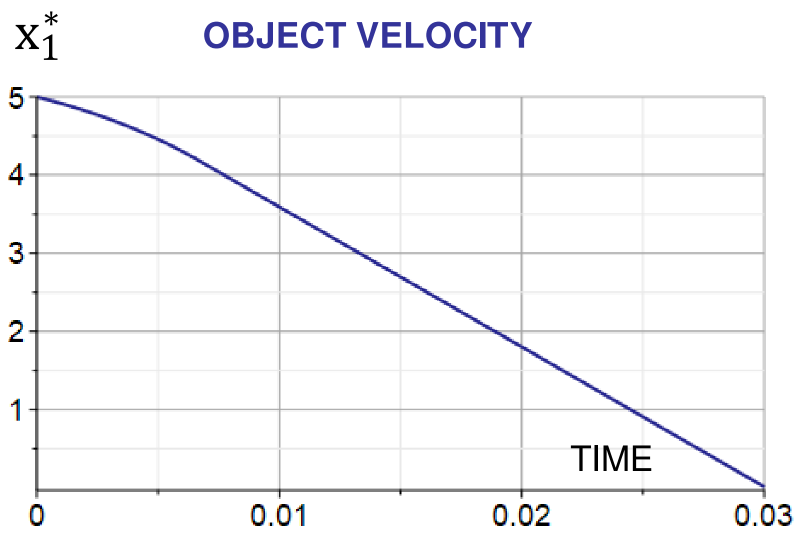


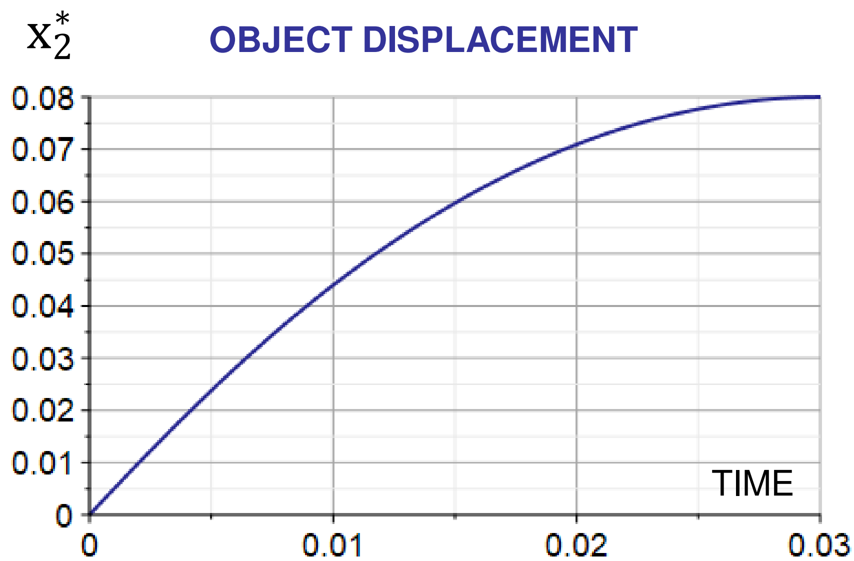
IV. Investigation of state-dependent formulation

C. Application to simple AIA problem - numerical results









IV. Investigation of state-dependent formulation

D. More complex AIA problem - formulation

Valve area as a control variable:

$$x = [\dot{u}, u, m]^T = [x_1, x_2, x_3]^T, \quad x_0 = [v_0, 0, m_0]^T, \quad \bar{u} = A_v(t)$$

Problem considered: Minimize: $J(x, \bar{u}) = \int_0^{t_f} \left(\frac{x_3 RT}{M(d - x_2)} - \frac{p_{ext} A}{M} - \frac{x_1^2}{2(d - x_2)} \right)^2 dt$

With respect to: $\bar{u} \in \langle 0, A_V^{max} \rangle$, Subject to: $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{x_3 RT}{M(d - x_2)} + \frac{p_{ext} A}{M} \\ x_1 \\ -\bar{u} \left(\frac{x_3 RT}{A(d - x_2)} - p_{ext} \right) \end{bmatrix}$

Hamiltonian: $H(x, \lambda, \bar{u}, t) = \left(-\frac{x_3 RT}{M(d - x_2)} + \frac{p_{ext} A}{M} \right) \lambda_1 + x_1 \lambda_2 - \bar{u} \left(\frac{x_3 RT}{A(d - x_2)} - p_{ext} \right) \lambda_3$

$$+ \left[\left(\frac{x_3 RT}{M(d - x_2)} - \frac{p_{ext} A}{M} \right) - \frac{x_1^2}{2(d - x_2)} \right]^2$$

Optimality conditions:

1. The inequality $H^* \leq H$ yields: $\bar{u}^* = \begin{cases} \bar{u}_{min} \text{ for } \lambda_3^* < 0 \\ \bar{u}_{max} \text{ for } \lambda_3^* > 0 \end{cases}$

2. Singular arc: $\frac{\partial H}{\partial \bar{u}} = 0 \rightarrow \lambda_3^* = 0$, finding control function $-$ nontrivial

IV. Investigation of state-dependent formulation

D. More complex AIA problem - proposed solution method

Remarks:

1. $H(t) = \text{const.}$: i) at singular arc, ii) when $\bar{u}^* = u_{min}$ or $\bar{u}^* = u_{max}$

   $H(t) \neq 0$ for $t \in \langle 0, t_f \rangle$ due to possible control discontinuities.

2. The last stage follows optimal state-dependent path: $x_3 = \frac{M(x_1^*)^2}{2RT} + \frac{p_{ext}A(d-x_2^*)}{RT}$

   $\rightarrow$ control $\bar{u}^* = \bar{u}^*(x_1, x_2)$ at singular arc with $\lambda_1^*(t) = \lambda_2^*(t) = \lambda_3^*(t) = 0$.

- The proposed approach:

  Find $\{\lambda_1^*(0), \lambda_2^*(0), \lambda_3^*(0)\}$ using the condition of reaching singular arc

- Alternative solution:

  a. Assumption of control strategy with three-stages:

  i) min. valve opening, i) max. valve opening, iii) following state-dependent path

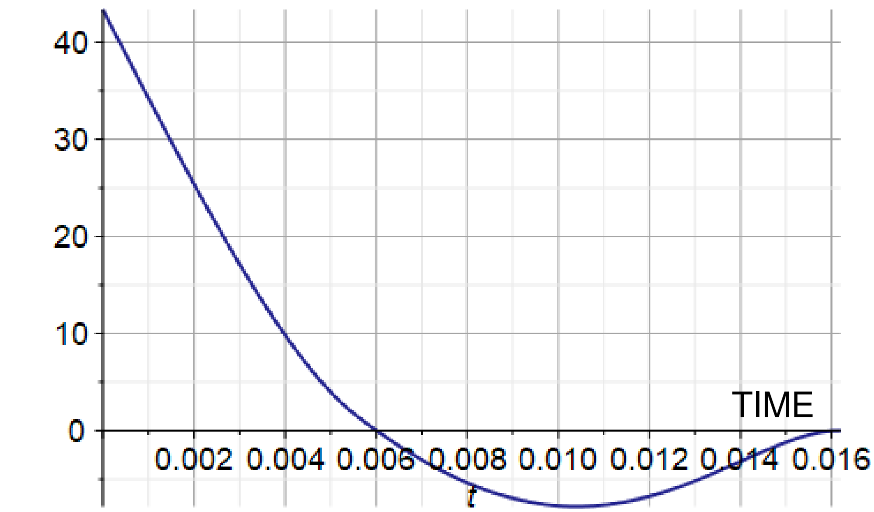
  b. Optimization of control switching times.

  c. Verification of Pontryagin's optimality conditions.

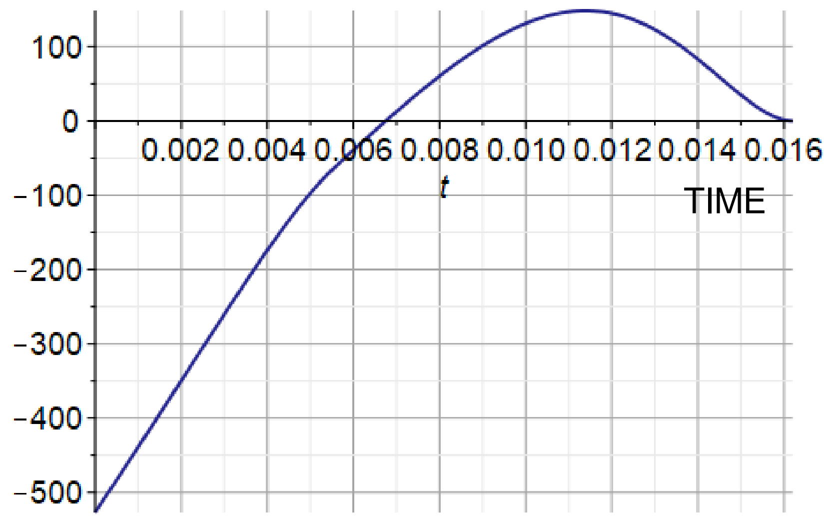
IV. Investigation of state-dependent formulation

D. More complex AIA problem - numerical results

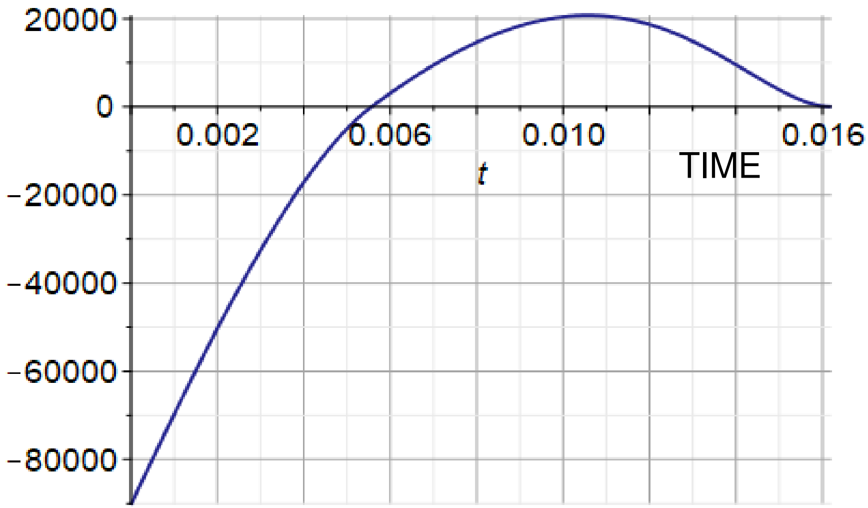
λ_1^* **THE FIRST COSTATE**



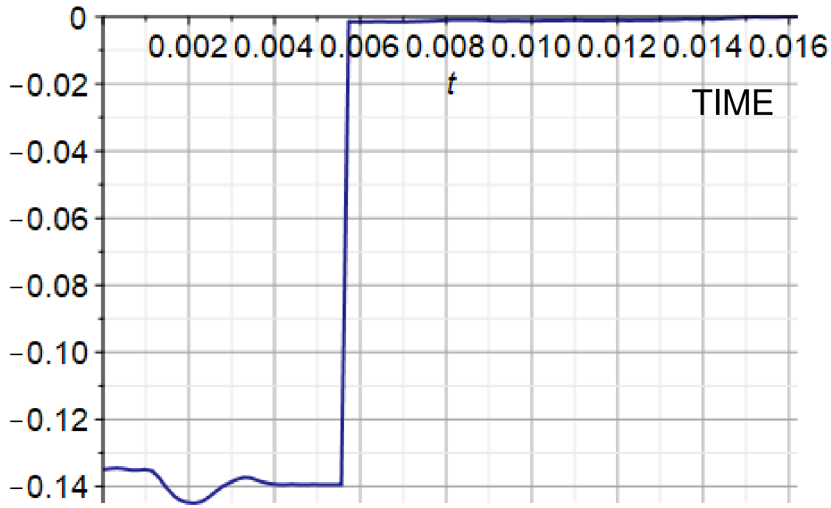
λ_2^* **THE SECOND COSTATE**



λ_3^* **THE THIRD COSTATE**

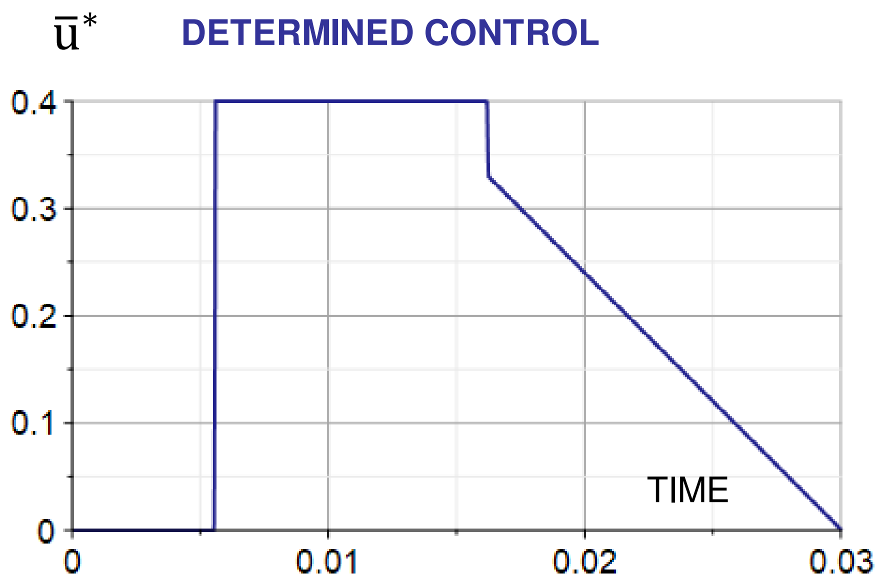


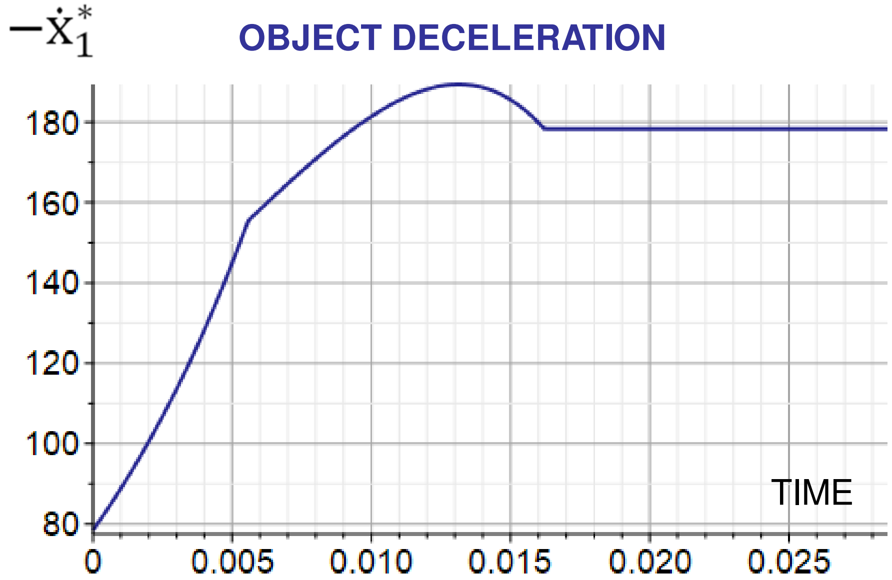
H^* **HAMILTONIAN**

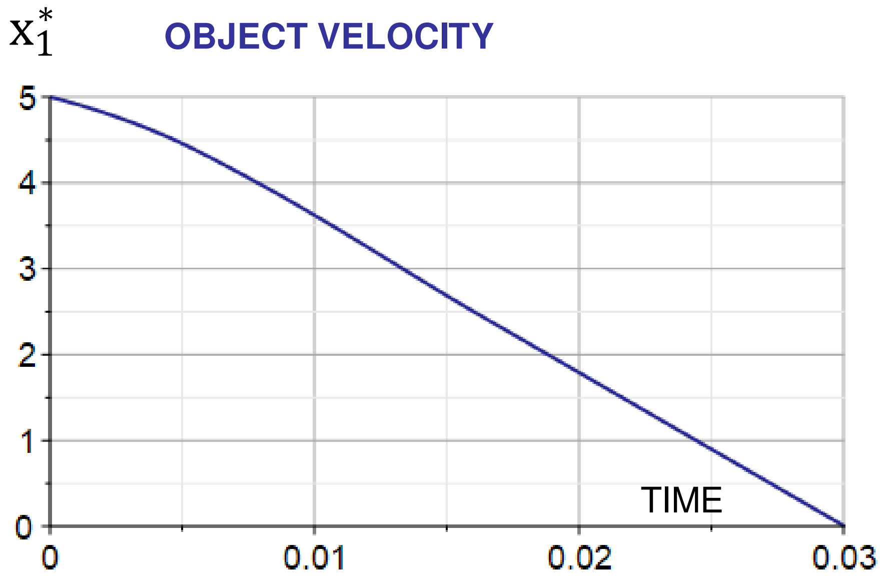


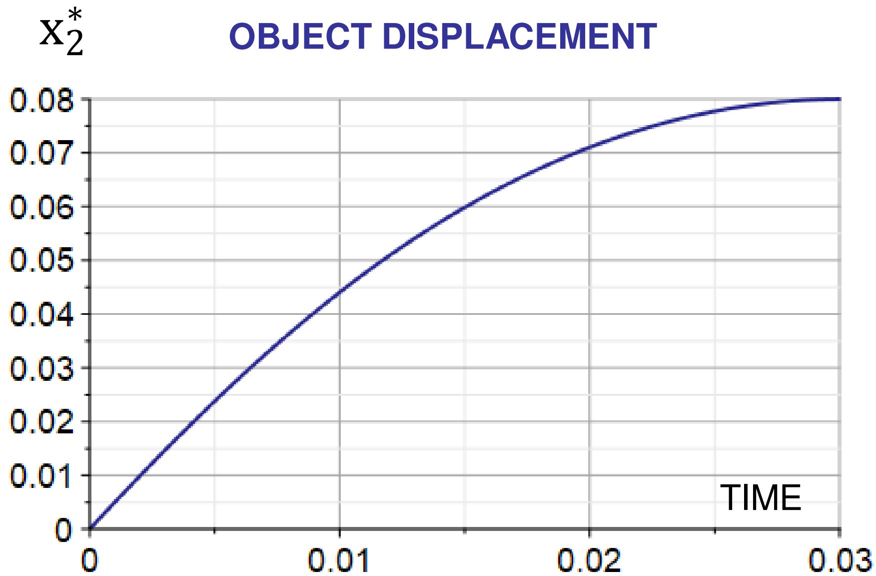
IV. Investigation of state-dependent formulation

D. More complex AIA problem - numerical results









IV. Investigation of state-dependent formulation

E. Application of direct discretization methods

<u>The concept of direct methods</u>:

1. Discretization of the control function in time domain.
2. Application of selected time integration method
   - transformation into large nonlinear optimization problem
3. Application of algorithmic differentiation to construct gradients and Hessians
4. Application of numerical optimization methods to find discrete values of the control function

CasADi

CasADi: a software framework for nonlinear optimization and optimal control

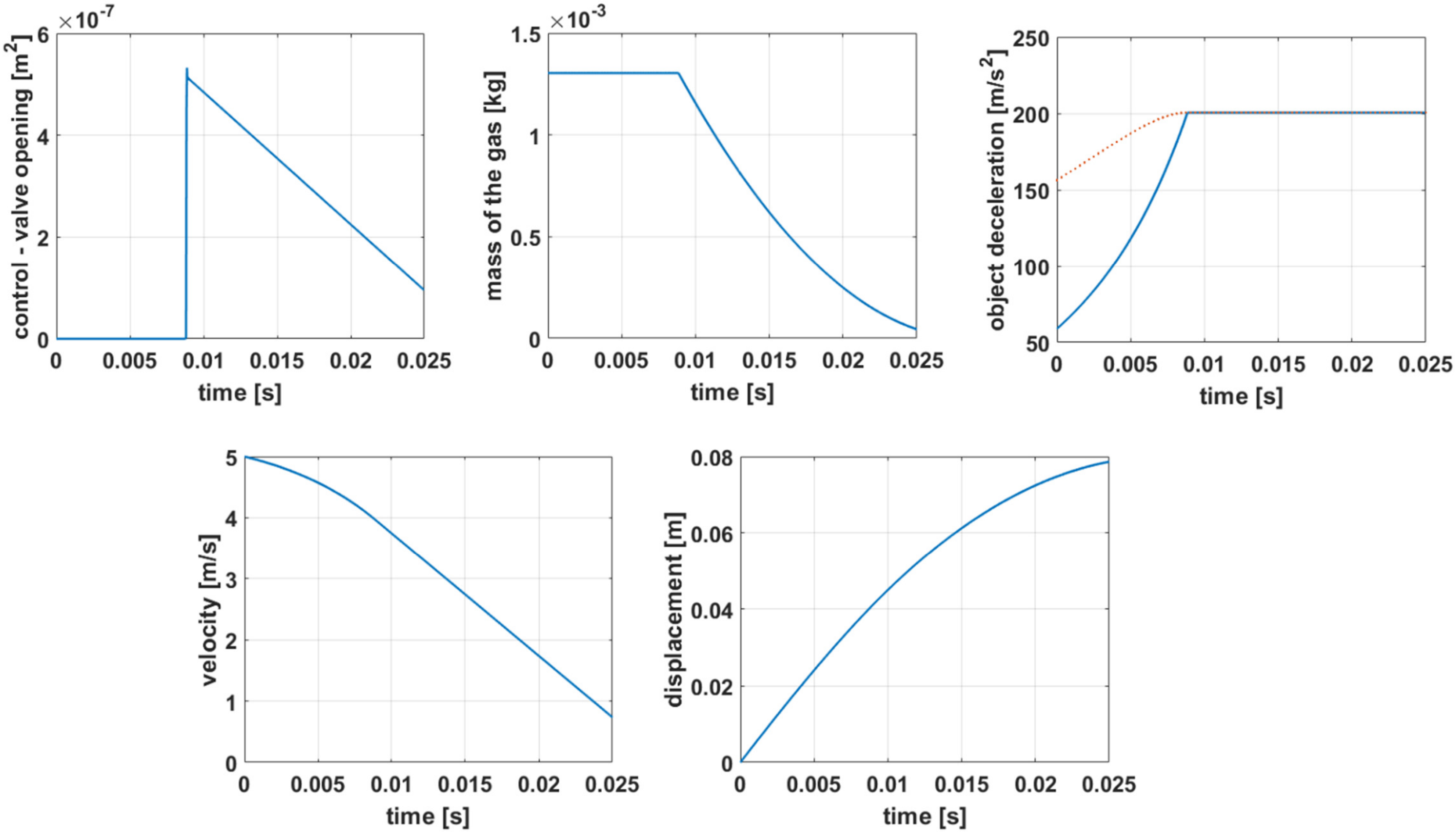
Joel A. E. Andersson, Joris Gillis, Greg Horn, James B. Rawlings & Moritz Diehl

Mathematical Programming Computation **11**, 1–36(2019) | Cite this article

IV. Investigation of state-dependent formulation

F. Numerical example: single-chamber absorber

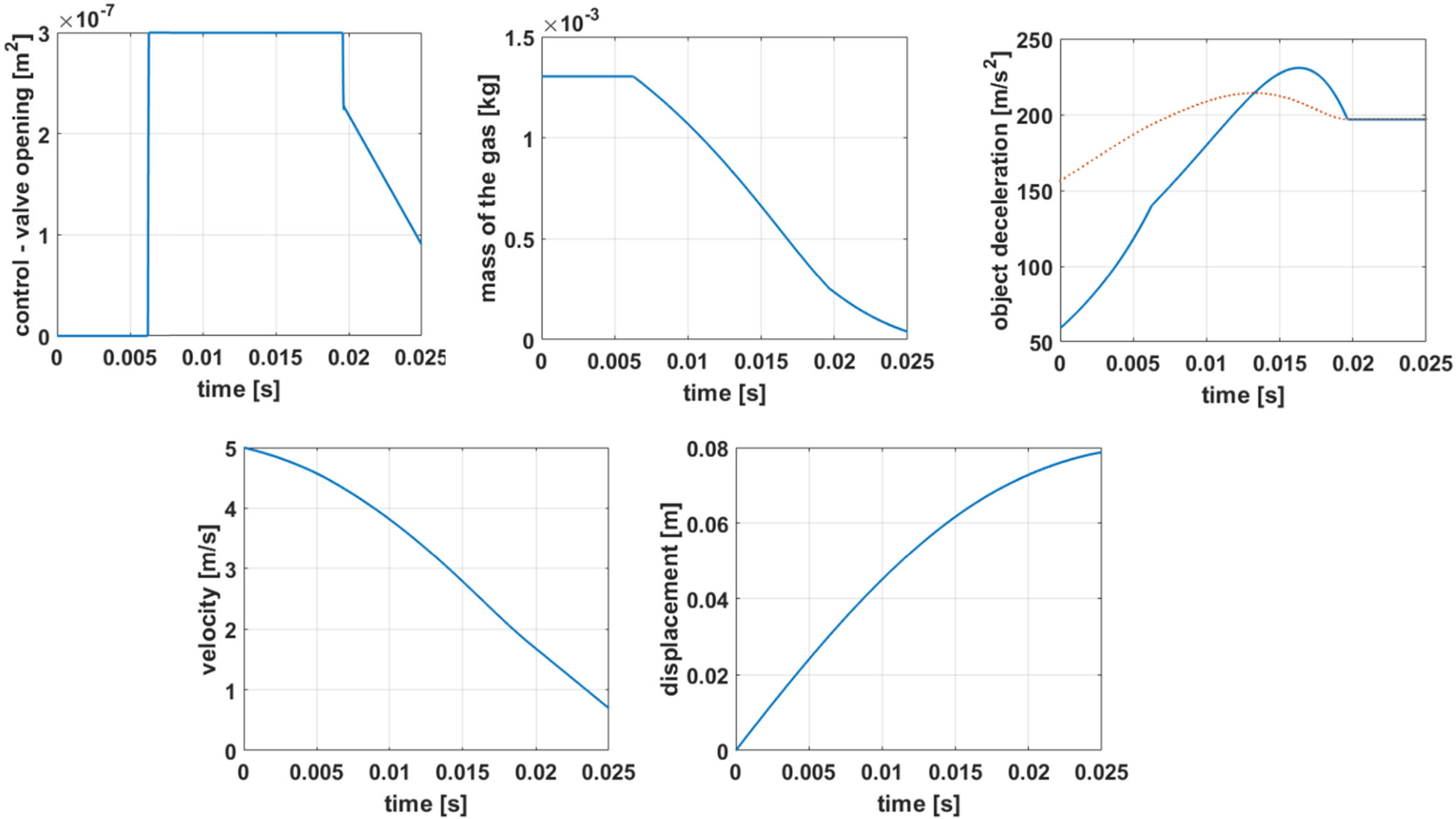
Control of valve opening - **no constraints**



IV. Investigation of state-dependent formulation

F. Numerical example: single-chamber absorber

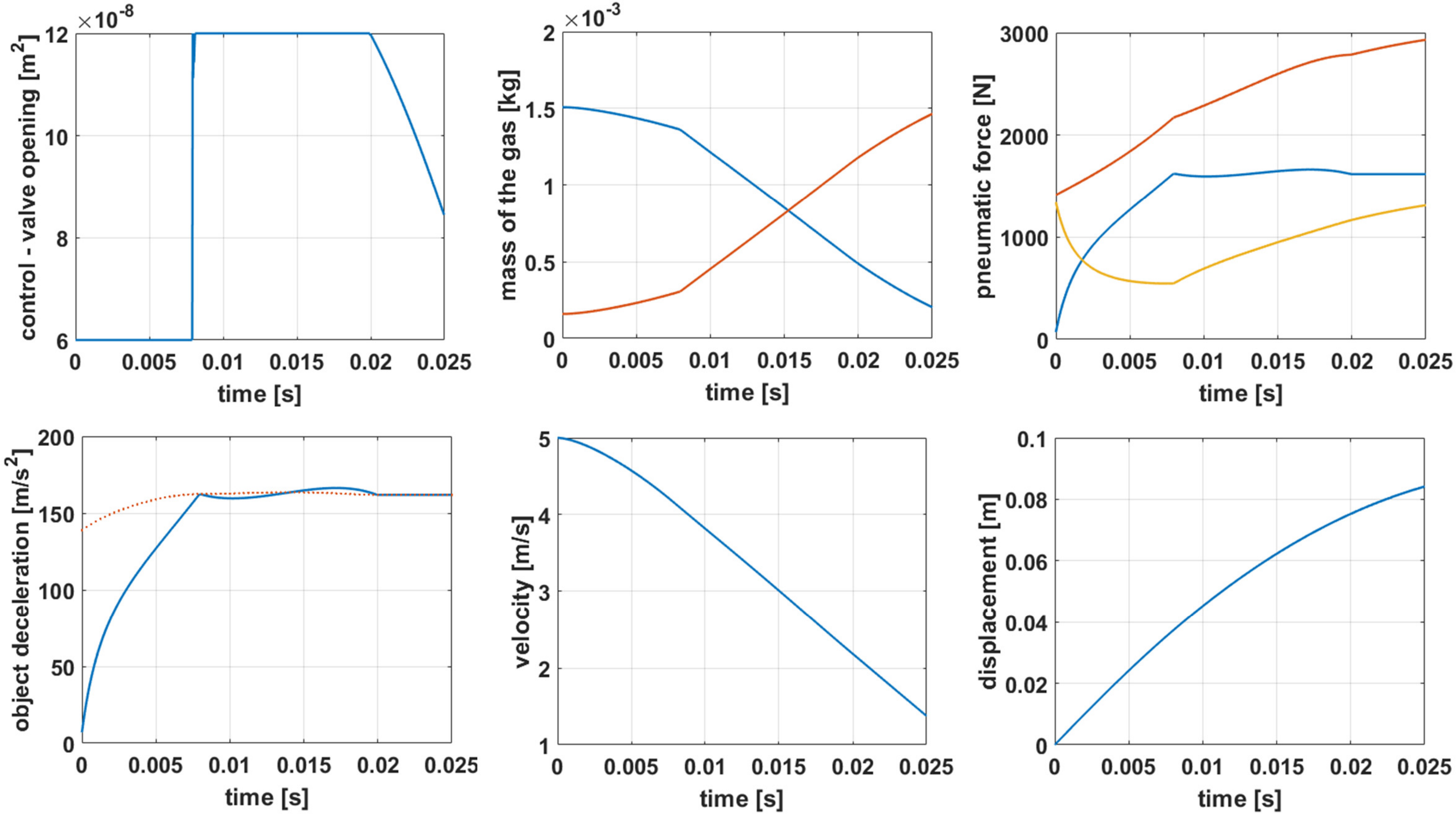
Control of valve opening - **active constraints**



IV. Investigation of state-dependent formulation

G. Numerical example: double-chamber absorber

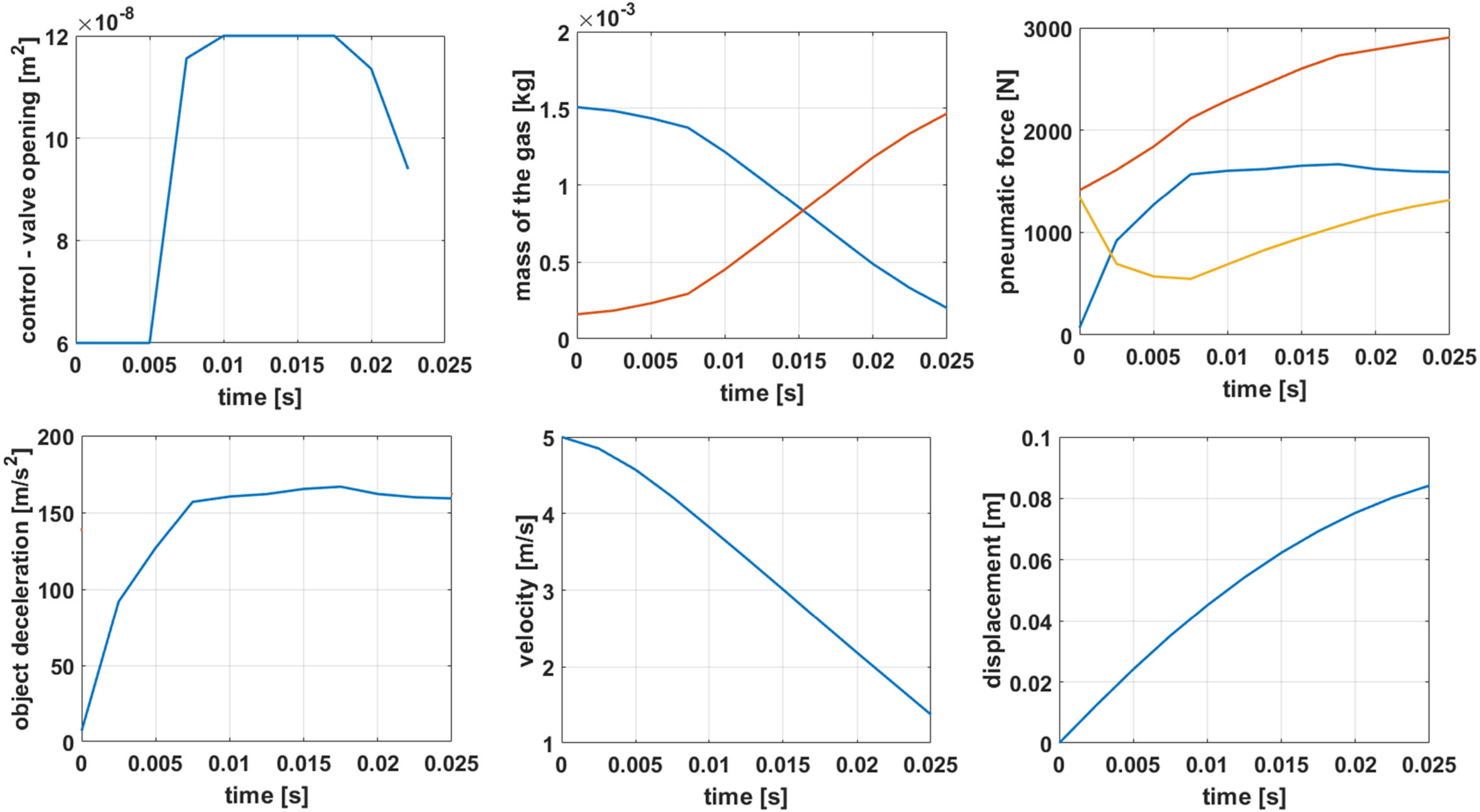
Control of valve opening - **large number of steps**



IV. Investigation of state-dependent formulation

G. Numerical example: double-chamber absorber

Control of valve opening - **small number of steps**



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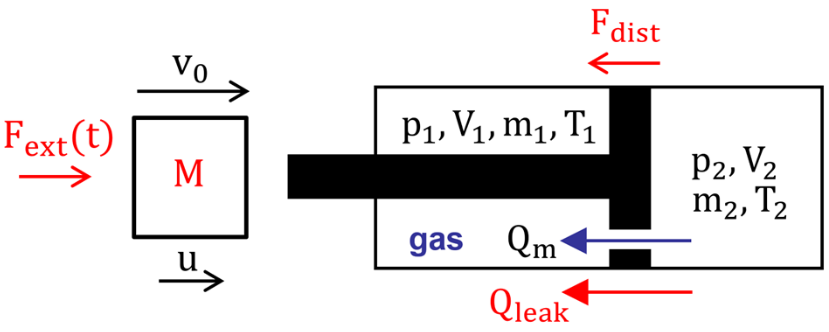
- construction of predictive model
- time-continuous approach
- control function and response parametrisation

VI. **Conclusions**

V. Adaptive and predictive control methods

A. Control problem for system with disturbances

- Model of system with disturbances:



$$M\ddot{u} + F_p + F_{dist}(t) = F_{ext}(t) \tag{1}$$

$$\dot{m}_1 = Q_m\left(p_1, p_2, T_{up}^c, C_v A_v\right) + Q_{leak}(t) \tag{2}$$

$$\dot{m}_2 = -Q_m\left(p_1, p_2, T_{up}^c, C_v A_v\right) - Q_{leak}(t) \tag{3}$$

$$\dot{Q}_1 + Q_m c_p T_{up}^c + Q_{leak}(t) c_p T_{up}^d = \tfrac{d}{dt}(m_1 c_v T_1) + p_1 \dot{V}_1 \tag{4}$$

$$\dot{Q}_2 - Q_m c_p T_{up}^c - Q_{leak}(t) c_p T_{up}^d = \tfrac{d}{dt}(m_2 c_v T_2) + p_2 \dot{V}_2 \tag{5}$$

$$T_v \frac{dA_v}{dt} + A_v = k_v u_v(t) \tag{6}$$

Features:

- force disturbance
- gas leakage disturbance
- unknown parameters
- control system equation

$A_v \geq 0$ - semi-active system

$A_v < 0$ - active system

- Two versions of state-dependent path-tracking problem:

$$\text{Minimize:} \quad \int_0^T \left(F_p\left(u_v(t)\right) - \frac{M\dot{u}(t)^2}{2\left(d - u(t)\right)} - \left[F_{ext}(t) - F_{dist}(t)\right]\right)^2 + qA_v\left(u_v(t)\right)^2 \, dt$$

$$\text{or Minimize:} \quad \int_0^T \left(\ddot{u}\left(u_v(t)\right) - \frac{\dot{u}(t)^2}{2\left(d - u(t)\right)}\right)^2 + qA_v\left(u_v(t)\right)^2 \, dt \qquad q = \begin{cases} 0 \text{ for } A_v \geq 0 \\ \bar{q} \text{ for } A_v < 0 \end{cases}$$

V. Adaptive and predictive control methods

B. Construction of the predictive model

- Assumptions for model derivation:
  - joint prediction of external and disturbance force: $F_{ext\_dist}(t)$,
  - neglecting heat transfer from the chambers: $\dot{Q}_1 = \dot{Q}_2 = 0$

- Applied mathematical transformations:

$$\int_{t_0}^{t} (\text{Eq. }2 + \text{Eq. }3)\, dt = 0$$

$$\left[\begin{array}{l} \displaystyle\int_{u_0}^{u} (\text{Eq. }1)\, du = 0 \\ \displaystyle\int_{t_0}^{t} (\text{Eq. }4 + \text{Eq. }5)\, dt = 0 \end{array}\right.$$

$$\int_{t_0}^{t} (\text{Eq. }5)\, dt = 0$$

$$\Rightarrow$$

$$M\ddot{u} + F_p - F_{ext\_dist}(t) = 0$$

$$\dot{m}_1 = Q_m(p_1, p_2, T_2, C_v A_v) + Q_{leak}(t)$$

$$\left.\begin{array}{c} m_1 + m_2 = m \\ \displaystyle\frac{1}{2}M(v_0^2 - v^2) + \int_{u_0}^{u} F_{ext\_dist}(t)du \;=\; \Delta U_1 + \Delta U_2 \\ \displaystyle\frac{p_2 V_2^{\kappa}}{m_2^{\kappa}} = \frac{p_2^0 (V_2^0)^{\kappa}}{(m_2^0)^{\kappa}} \end{array}\right] \text{algebraic equations}$$

$$T_v \frac{dA_v}{dt} + A_v = k_v u_v(t)$$

- Features of the predictive model:

control signal $\xrightarrow{\text{differential equations}}$ system kinematics

system kinematics $\xrightarrow{\text{algebraic equations}}$ control signal

V. Adaptive and predictive control methods

C. System identification

- Identification of system parameters and disturbances

Equation of motion: $M\ddot{u} + F_p(p_1, p_2) = F_{ext}(t)\text{-}F_{dist}(t)$

Approach 1: assumption of specific form of force, e.g.: $F_{dist} = ku^p + c\dot{u}^r + d\ddot{u}^s$

- Direct identification: no. of unknowns = no. time instants

Case: $F_{dist} = ku + c\dot{u}$

$$\begin{bmatrix} \ddot{u}_1 & \dot{u}_1 & u_1 \\ \ddot{u}_2 & \dot{u}_2 & u_2 \\ \ddot{u}_3 & \dot{u}_3 & u_3 \end{bmatrix} \begin{bmatrix} M \\ c \\ k \end{bmatrix} + \begin{bmatrix} F_p^1 \\ F_p^2 \\ F_p^3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} M \\ c \\ k \end{bmatrix} = -\begin{bmatrix} \ddot{u}_1 & \dot{u}_1 & u_1 \\ \ddot{u}_2 & \dot{u}_2 & u_2 \\ \ddot{u}_3 & \dot{u}_3 & u_3 \end{bmatrix}^{-1} \begin{bmatrix} F_p^1 \\ F_p^2 \\ F_p^3 \end{bmatrix}$$

Solution depends on measurements times and conditioning of the main matrix

- Optimization approach:
  (larger no. time instants)

$$\text{Minimize: } \sum_{i=1}^{n} \left(M\ddot{u}_i + c\dot{u}_i + ku_i + F_p^i \right)^2$$

- Inertial force ident.: $F_{ext} = \dfrac{F_{ext}^{(i)}}{\ddot{u}^{(i)}} \ddot{u}$ $\rightarrow$ $\underbrace{\left(M - \dfrac{F_{ext}^{(i)}}{\ddot{u}^{(i)}} \right)}_{M_{eq}} \ddot{u}^{(i)} + F_p^i = 0$ $\rightarrow$ $\boxed{M_{eq} = -\dfrac{F_p^{(i)}}{\ddot{u}^{(i)}}}$

V. Adaptive and predictive control methods

C. System identification

- Identification of system parameters and disturbances

Equation of motion: $M\ddot{u} + F_p(p_1, p_2) = F_{ext}(t)\text{-}F_{dist}(t)$

Approach 2: direct identification at given time instants

- System of equations:

$$\begin{cases} M\ddot{u}^{(i)} + F_p^{(i)} + F_{dist}^{(i)} = 0, \qquad M\ddot{u}^{(i+1)} + F_p^{(i+1)} + F_{dist}^{(i+1)} = 0 \\ \frac{1}{2}M\left(v_{(i)}^2 - v_{(i+1)}^2\right) + \int_{u_{(i)}}^{u_{(i+1)}} F_{dist}(u)du = \left(U_1^{(i+1)} + U_2^{(i+1)}\right) - \left(U_1^{(i)} + U_2^{(i)}\right) \end{cases}$$

- requires assuming disturbance change, - effective for short control steps
- extrapolation to entire process: $\bar{F}_{dist}(t_i, t) = f_i(F_{dist}(t_1), F_{dist}(t_2), \dots, F_{dist}(t_i), t)$

Equation of mass balance: $\dot{m}_1 = Q_m\left(p_1, p_2, T_{up}^c, C_v A_v\right) + Q_{leak}(t)$

$\left.\begin{array}{l}\text{Approach 1}\\ \text{Approach 2}\end{array}\right\}$ require considering actual valve opening

Parameters and disturbance identification → update of predictive model each control step

V. Adaptive and predictive control methods

D. Final formulation for problem with disturbances

- Problem solved at each step:

$$\text{Minimize:} \int_{t_i}^{T} \left(\ddot{u}(u_v(t)) + \frac{\dot{u}(t_i)^2}{2\left(d - u(t_i)\right)} \right)^2 + qA_v(u_v(t))^2 dt$$

$$\text{With respect to:} \quad u_v(t) \epsilon [u_{min}, u_{max}], \quad t \in (t_i, T)$$

$$\text{Subject to:} \quad M\ddot{u} + F_p - \bar{F}_{ext\_dist}(t_i, t) = 0$$

$$\dot{m}_1 = Q_m(C_v A_v) + \bar{Q}_{leak}(t_i, t)$$

$$T_v \frac{dA_v}{dt} + A_v = k_v u_v(t)$$

$$\int_{u(t_i)}^{u(T)} F_p du = \tfrac{1}{2} M\dot{u}(t_i)^2 + \int_{u(t_i)}^{u(T)} \bar{F}_{ext\_dist}(t_i, t) du$$

- Shortened prediction time:

$$\text{Minimize:} \int_{t_i}^{t_i + \Delta t} \left(\ddot{u}(u_v(t)) + \frac{\dot{u}(t_i)^2}{2\left(d - u(t_i)\right)} \right)^2 + qA_v(u_v(t))^2 dt$$

$$\text{With respect to:} \quad u_v(t) \epsilon [u_{min}, u_{max}], \quad t \in (t_i, t_i + \Delta t)$$

Subject to: actual predictive model

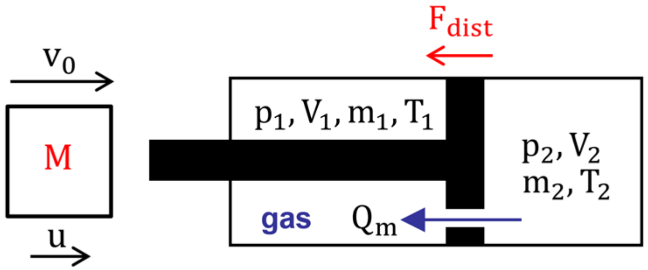
- Solution methods: i) optimal control with time-continuous approach

ii) control function parametrisation, iii) response parametrisation

V. Adaptive and predictive control methods

E. Optimal control (time-continuous approach)

- Standard problem: absorption of single-impact by semi-active system

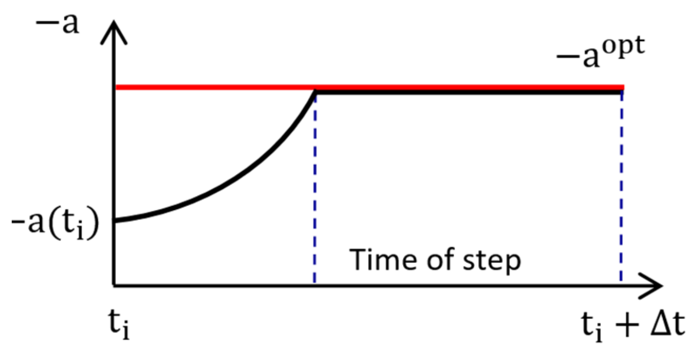


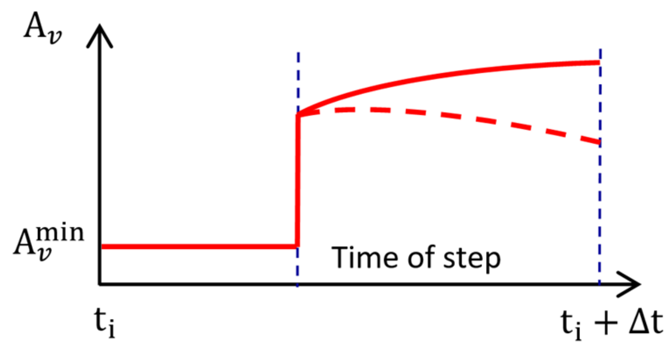
Objective: minim. of deceleration by valve area control
Case 1: unknown disturbance force
Case 2: unknown mass and disturbance force

- Mathematical formulation: $\text{Find } A_v^{opt}(t) = \arg\min \int_{t_i}^{t_i+\Delta t} \left(\ddot{u}(A_v(t)) + \frac{\dot{u}(t_i)^2}{2(d-u(t_i))} \right)^2 dt$

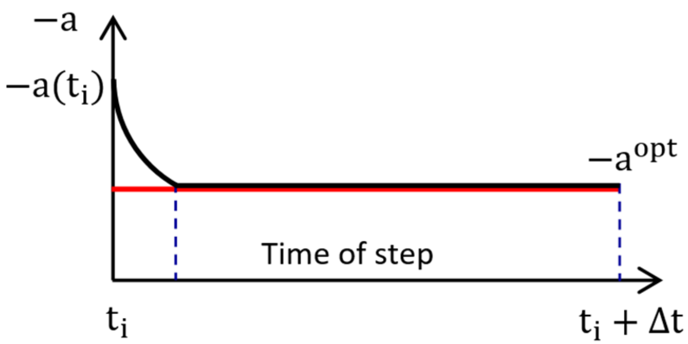
- Desired system operation near the optimal path:

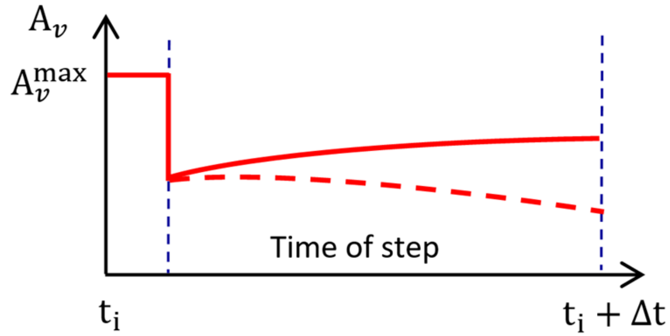
A. $-\ddot{u}(t_i) < \frac{\dot{u}(t_i)^2}{2(d-u(t_i))}$





B. $-\ddot{u}(t_i) > \frac{\dot{u}(t_i)^2}{2(d-u(t_i))}$



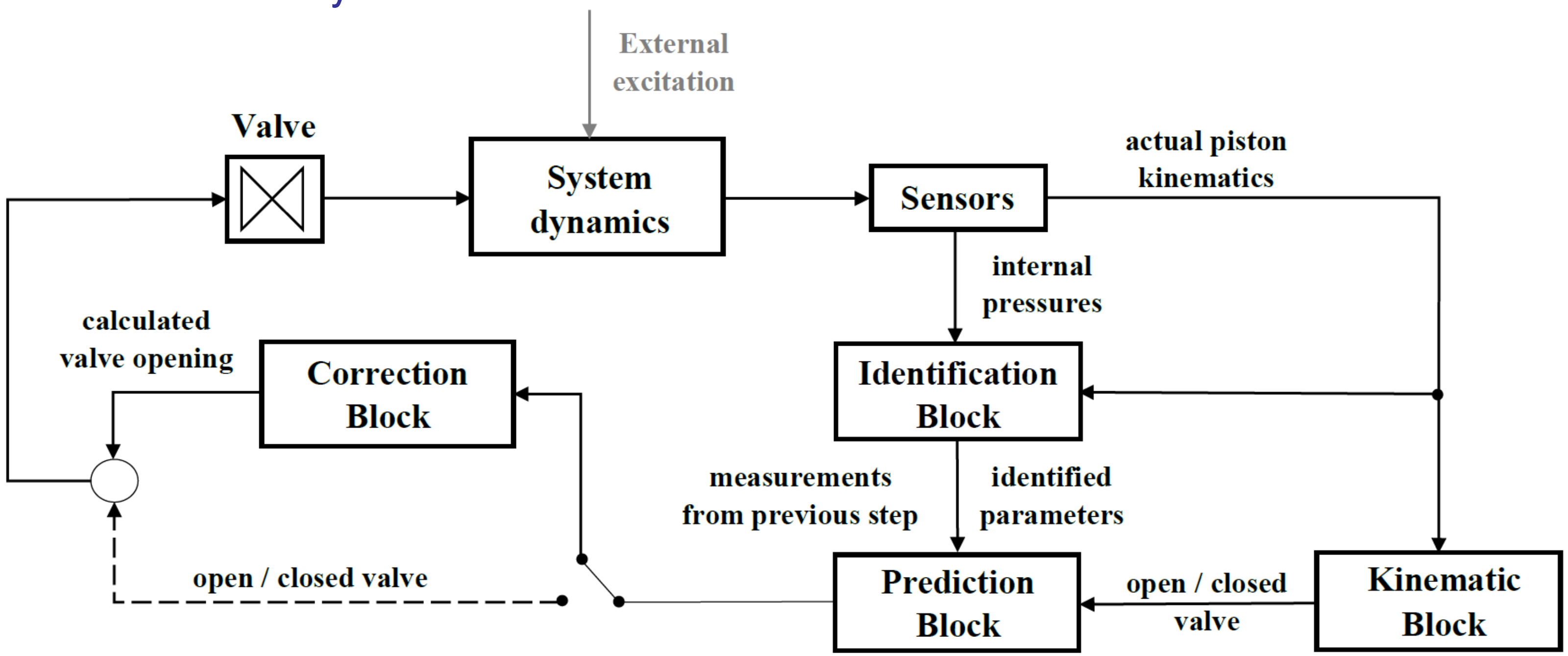


V. Adaptive and predictive control methods

E. Optimal control (time-continuous approach)

- Three-step control algorithm:
    1. System identification based on measurements and governing equations
    2. Prediction step: simulation of response with $\mathrm{A}_v(\mathrm{t}) = \mathrm{A}_v^{\mathrm{min}}$ or $\mathrm{A}_v(\mathrm{t}) = \mathrm{A}_v^{\mathrm{max}}$ to check if optimal deceleration $\mathrm{a}^{\mathrm{opt}}$ is reached
    3. Control determination step: calculation of valve opening $\mathrm{A}_v(\mathrm{t})$ giving $\ddot{\mathrm{u}}(\mathrm{t}) = \mathrm{a}^{\mathrm{opt}}$ using inverse dynamics prediction

- Scheme of the control system:

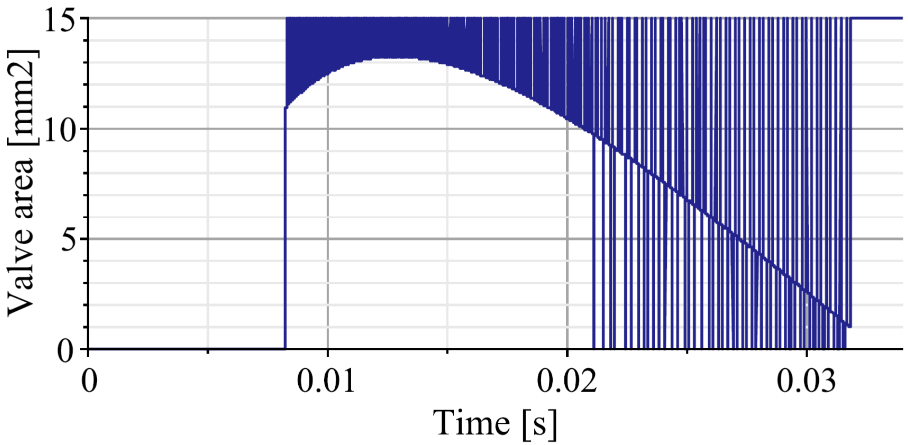


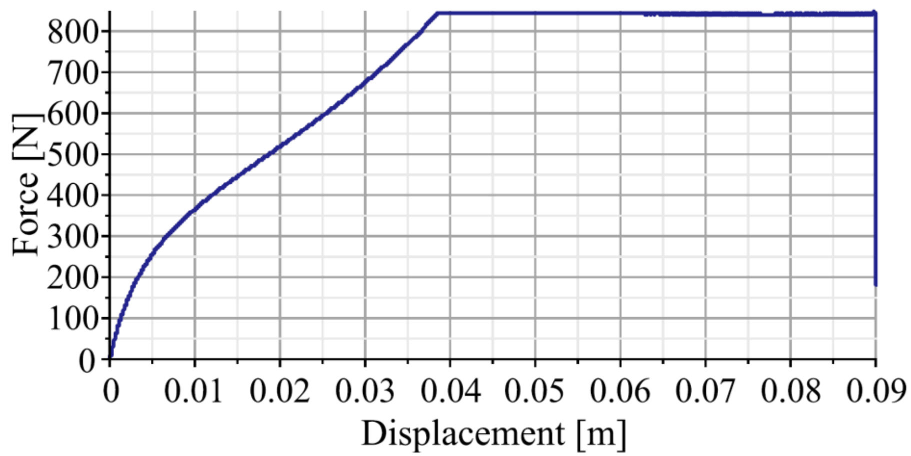
V. Adaptive and predictive control methods

E. Optimal control (time-continuous approach)

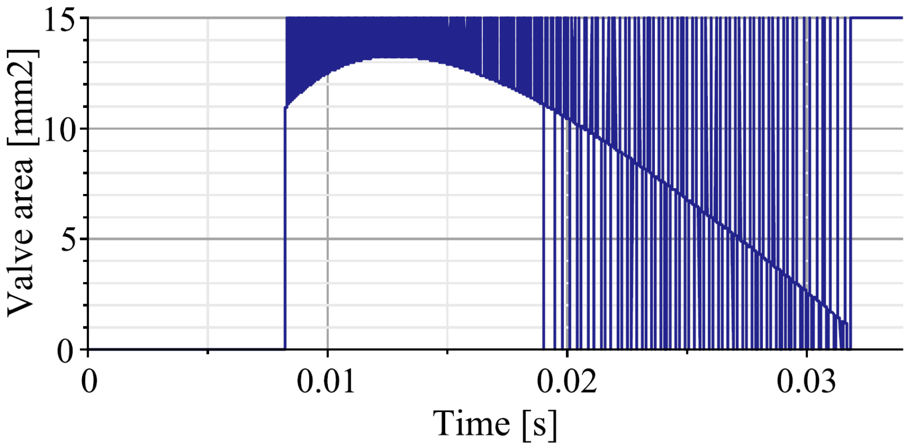
- The case of disturbance by <u>elastic force</u>

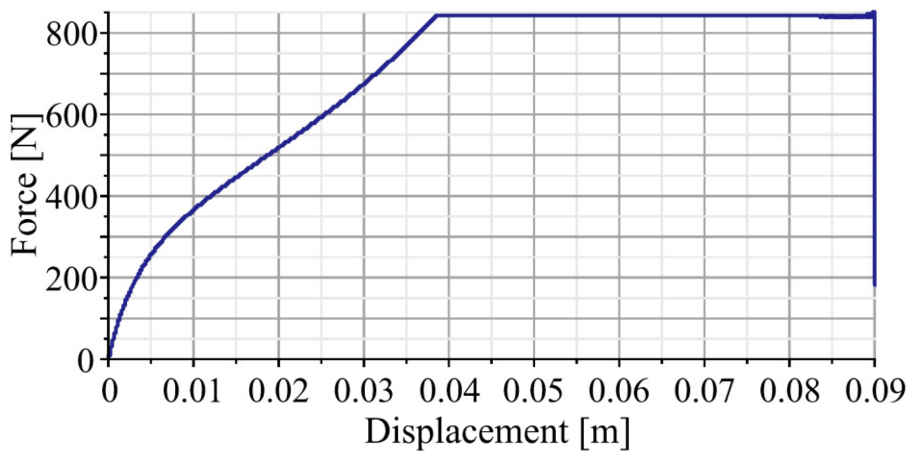
Unknown disturbance force





Unknown impacting object mass and disturbance force



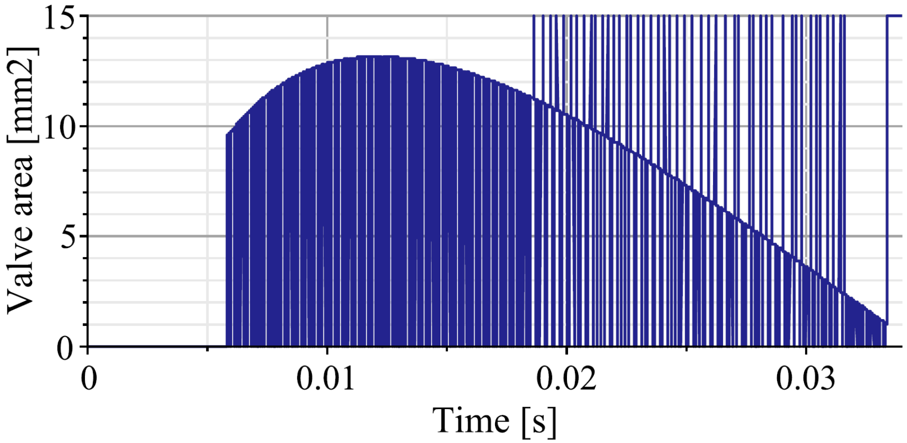


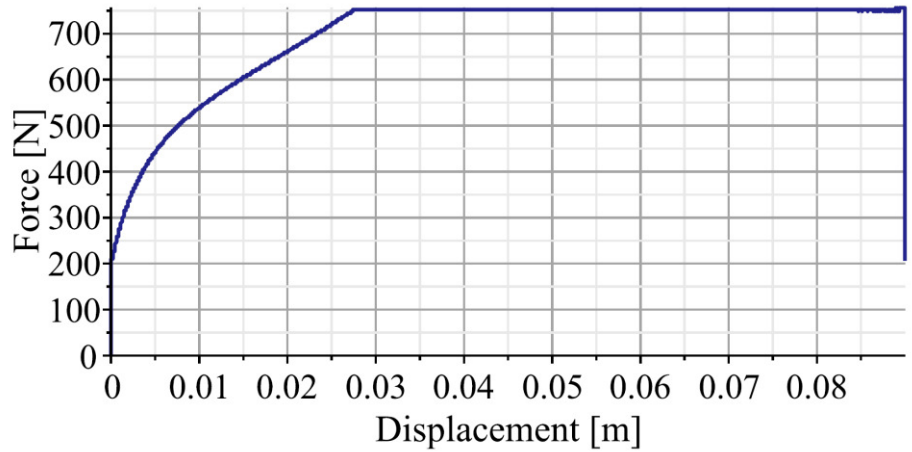
V. Adaptive and predictive control methods

E. Optimal control (time-continuous approach)

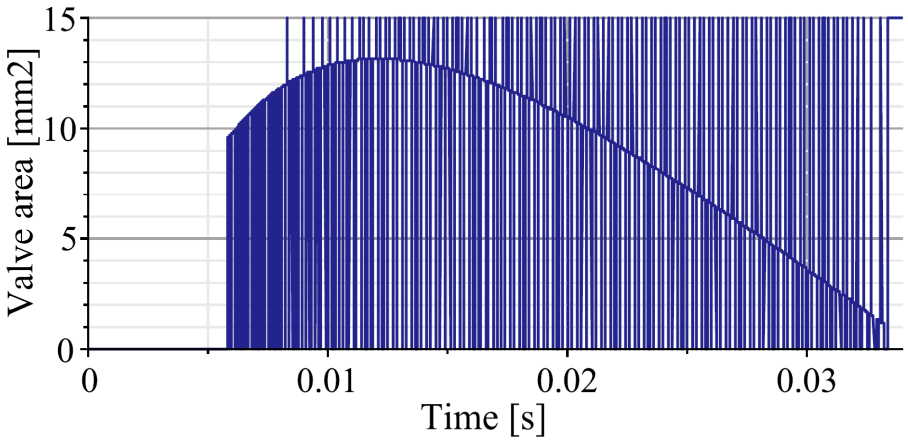
- The case of disturbance by viscous force

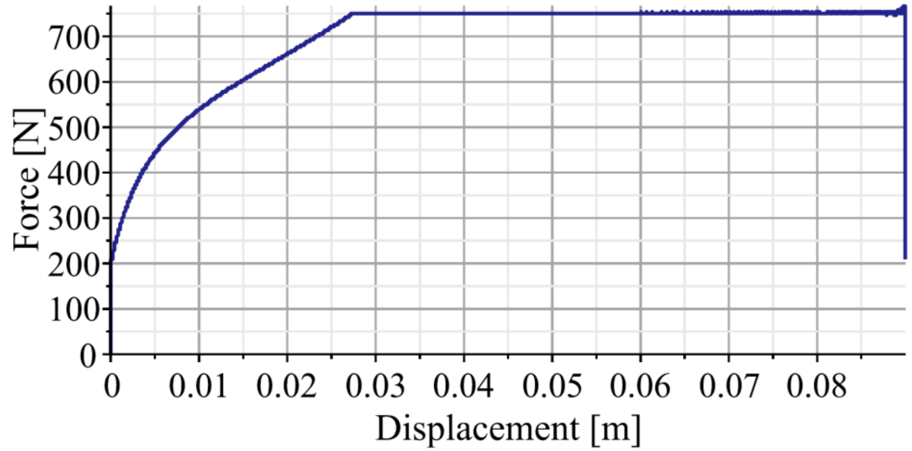
Unknown disturbance force





Unknown impacting object mass and disturbance force



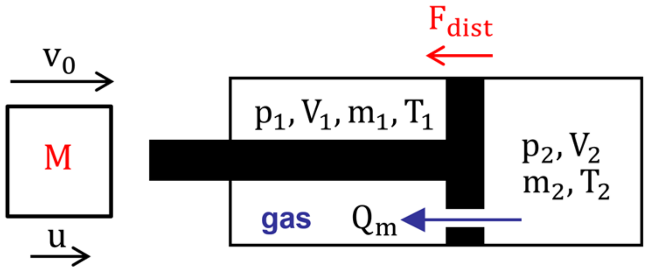


Graczykowski C., Faraj R., Identification-based predictive control of semi-active shock-absorbers for adaptive dynamic excitation mitigation, MECCANICA, Vol.55, No.12, pp.2571-2597, 2020

V. Adaptive and predictive control methods

F. Control function parameterisation

- Standard problem: absorption of single-impact by semi-active system



Objective: minim. of deceleration by valve area control

Features: unknown mass and disturbance force

- Mathematical formulation: Find $\boldsymbol{\beta}^{opt} = \arg\min \int_{t_i}^{t_i+\Delta t} \left(\ddot{u}(A_v(\boldsymbol{\beta}, t)) + \frac{\dot{u}(t_i)^2}{2(d - u(t_i))} \right)^2 dt$

- Simplest parametrization: $A_v(\boldsymbol{\beta}, t) = A_v^{const}$

1. Application of gradient-based methods

   Auxiliary function: $A_v^{opt}(t)$ – valve opening providing $\ddot{u}(t) = \ddot{u}(t_i)$, determined using IDP

   Starting point: $A_v^{ini}(t_i) = \frac{1}{2}[A_v^{opt}(t_i) + A_v^{opt}(t_i + \Delta t)]$

2. Application of linearized predictive model

   Solution using forward Euler method with one integration step

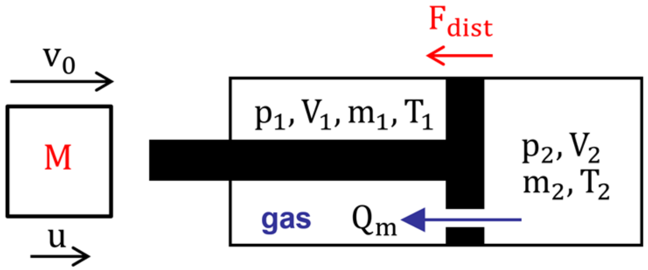
   $\ddot{u}(A_v^{const}, t) = f(p(t_i), m(t_i), T(t_i), u(t_i), v(t_i), a(t_i), A_v^{const}, t)$

   Analytical calculation of the objective function $I(A_v^{const})$ $\rightarrow$ $\boxed{A_v^{opt} = \arg\min I(A_v^{const})}$

V. Adaptive and predictive control methods

F. Control function parameterisation

- Standard problem: absorption of single-impact by semi-active system

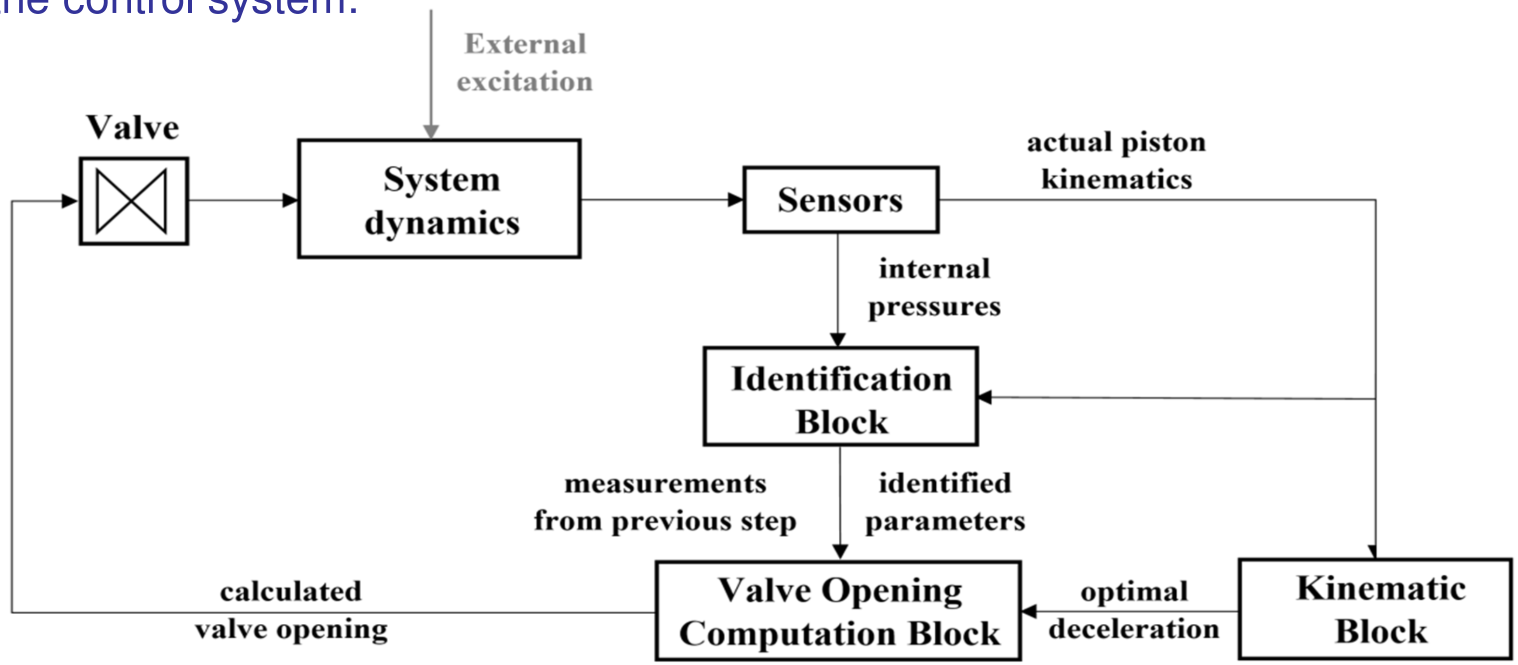


Objective: minim. of deceleration by valve area control

Features: unknown mass and disturbance force

- Mathematical formulation: Find $\boldsymbol{\beta}^{opt} = \arg\min \int_{t_i}^{t_i+\Delta t} \left(\ddot{u}(A_v(\boldsymbol{\beta}, t)) + \frac{\dot{u}(t_i)^2}{2(d - u(t_i))} \right)^2 dt$

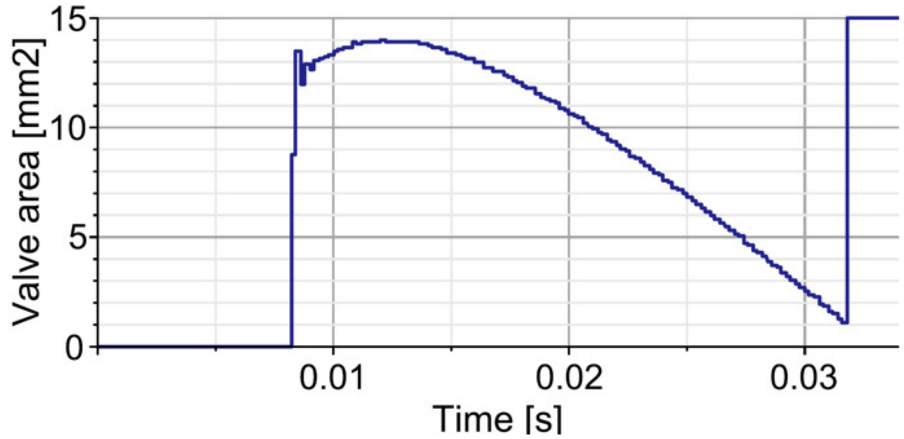
- Scheme of the control system:

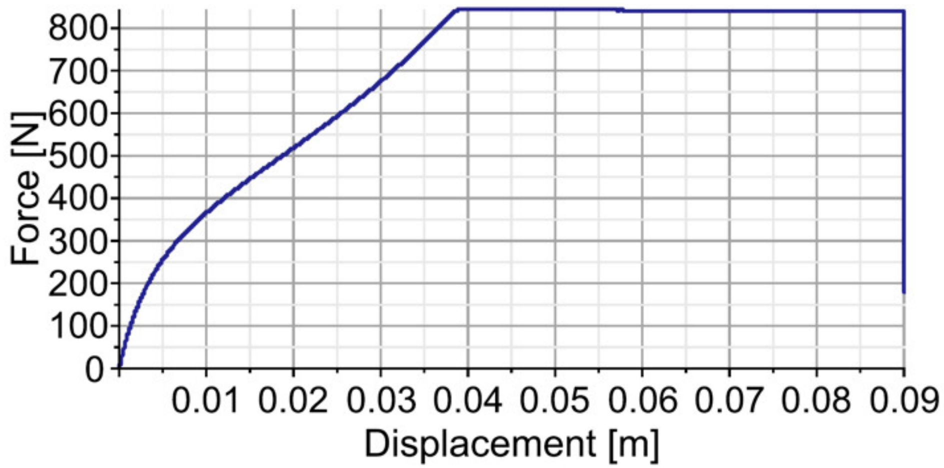


V. Adaptive and predictive control methods

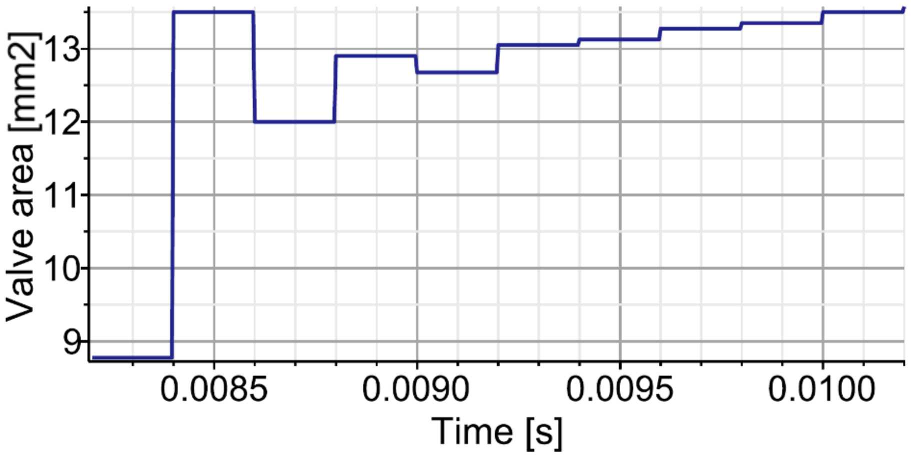
F. Control function parameterisation

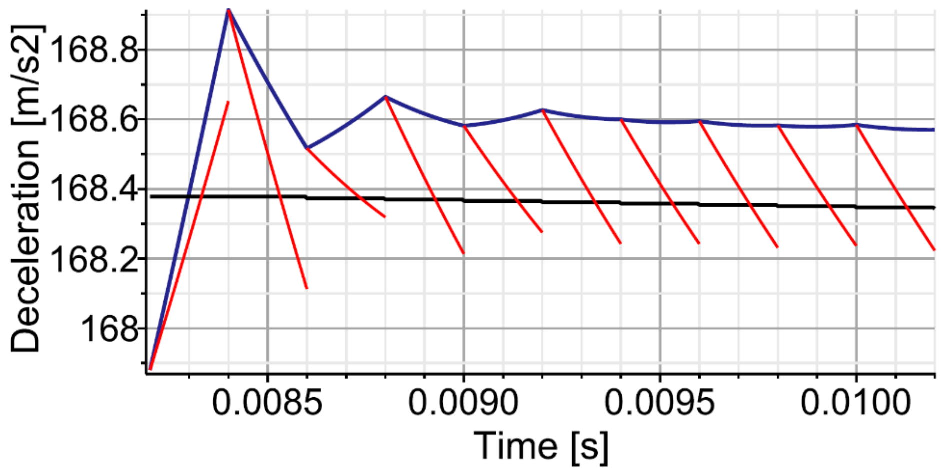
Operation during entire process





Details of method operation

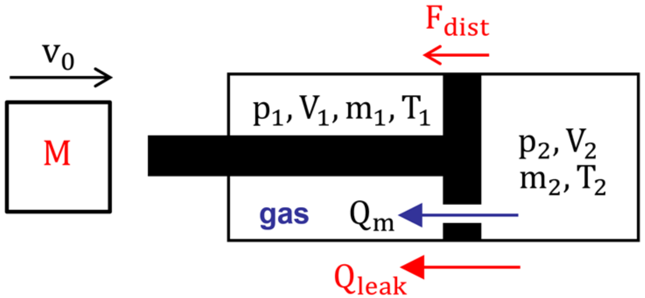




V. Adaptive and predictive control methods

F. Control function parameterisation

- Advanced problem: absorption of single-impact in system with leakage (semi-active / active control)



Objective: minim. of deceleration and operation cost by applied voltage control

Features: unknown mass, force disturbance and leakage

- Mathem. formulation: $\text{Find } \boldsymbol{\beta}^{opt} = \arg\min \int_{t_i}^{t_i+\Delta t} \left(\ddot{u}(u_v(\boldsymbol{\beta}, t)) + \frac{\dot{u}(t_i)^2}{2(d - u(t_i))} \right)^2 + qA_v(u_v(\boldsymbol{\beta}, t))^2 dt$

- Simplest parametrization: $u_v(\boldsymbol{\beta}, t) = u_v^{const}$

- Influence of voltage on valve opening area:

$$\boxed{T_v \frac{dA_v}{dt} + A_v = k_v u_v} \qquad u_v - \text{constant} \qquad k_v = \frac{A_v^{max}}{u_v^{max}} \text{ or } \frac{A_v^{min}}{u_v^{min}}$$

$$A_v(t) = A_v(t_i) e^{-\frac{(t-t_i)}{T_v}} + A_v^{max} \frac{u_v}{u_v^{max}} \left(1 - e^{-\frac{(t-t_i)}{T_v}}\right) \qquad \text{for } u_v \geq 0 \text{ and } k_v = \frac{A_v^{max}}{u_v^{max}}$$

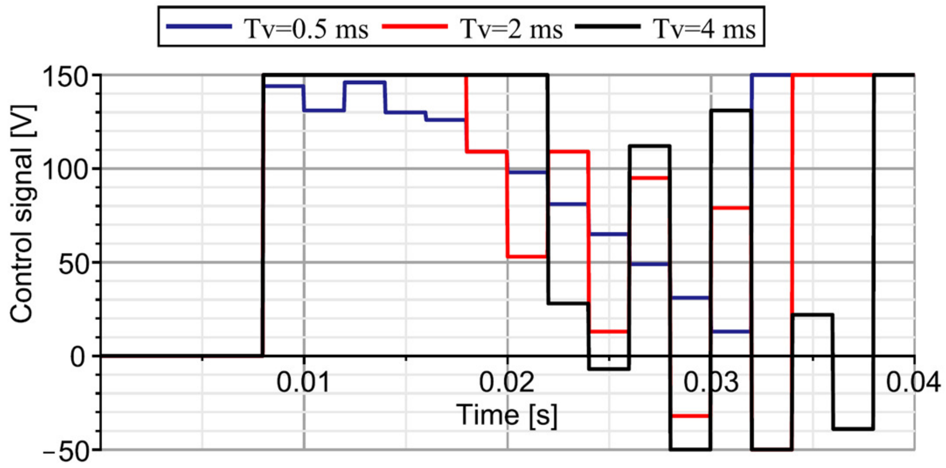
→ parametrisation of applied voltage corresponds to parametrisation of valve opening

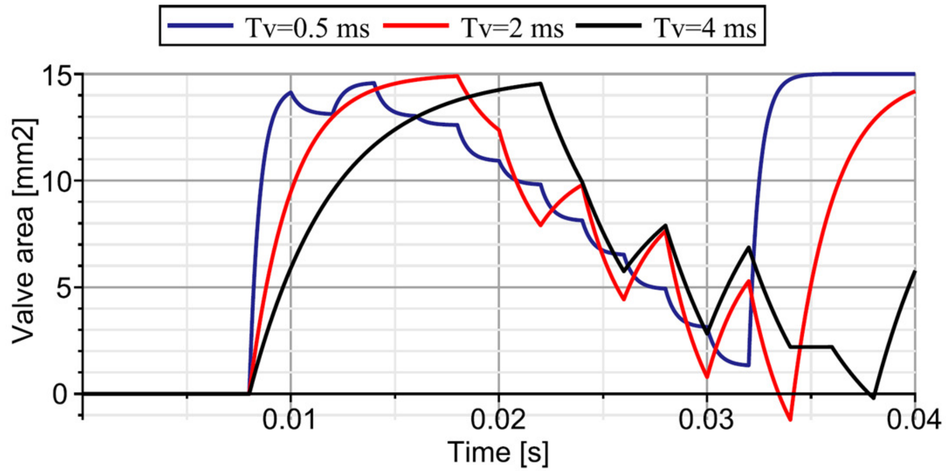
→ piecewise-constant voltage results in continuous but non-smooth valve opening

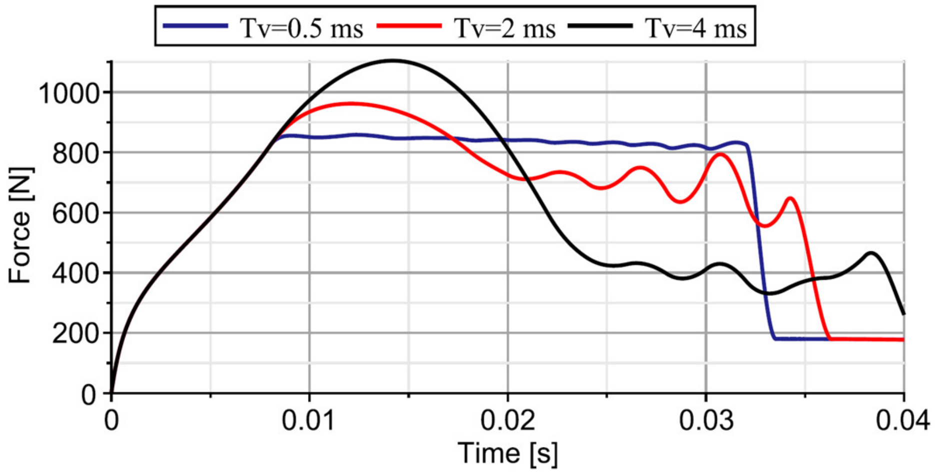
V. Adaptive and predictive control methods

F. Control function parameterisation

- Influence of time constant T_v on effectiveness of impact absorption process



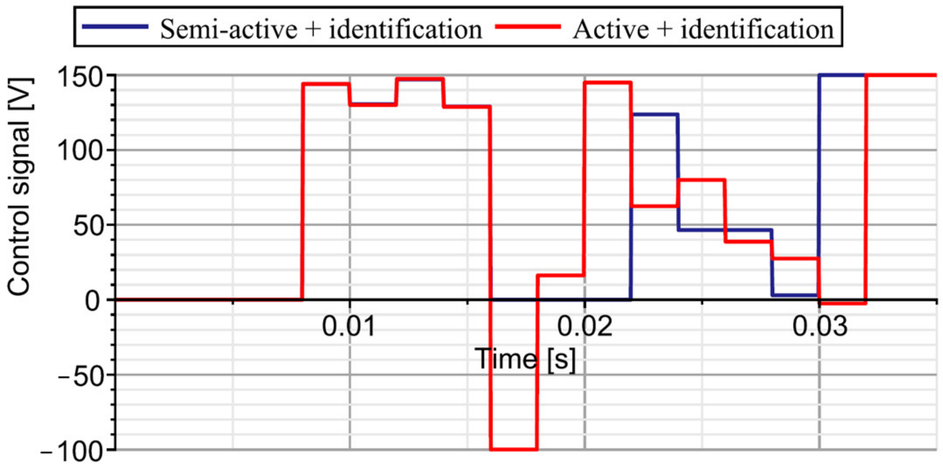


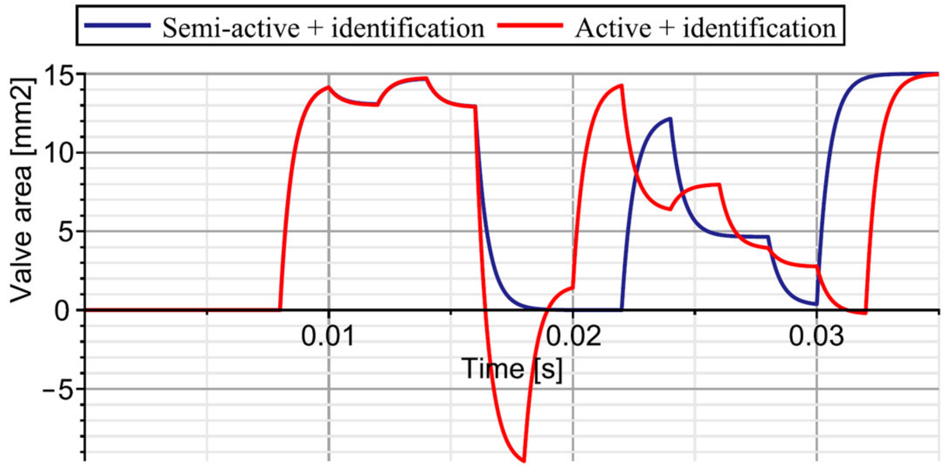


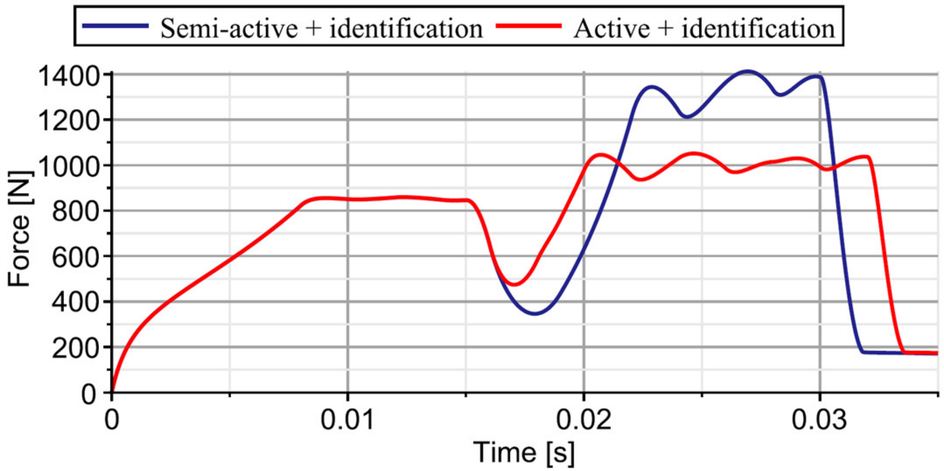
V. Adaptive and predictive control methods

F. Control function parameterisation

- Comparison of semi-active and active control in case of leakage disturbance



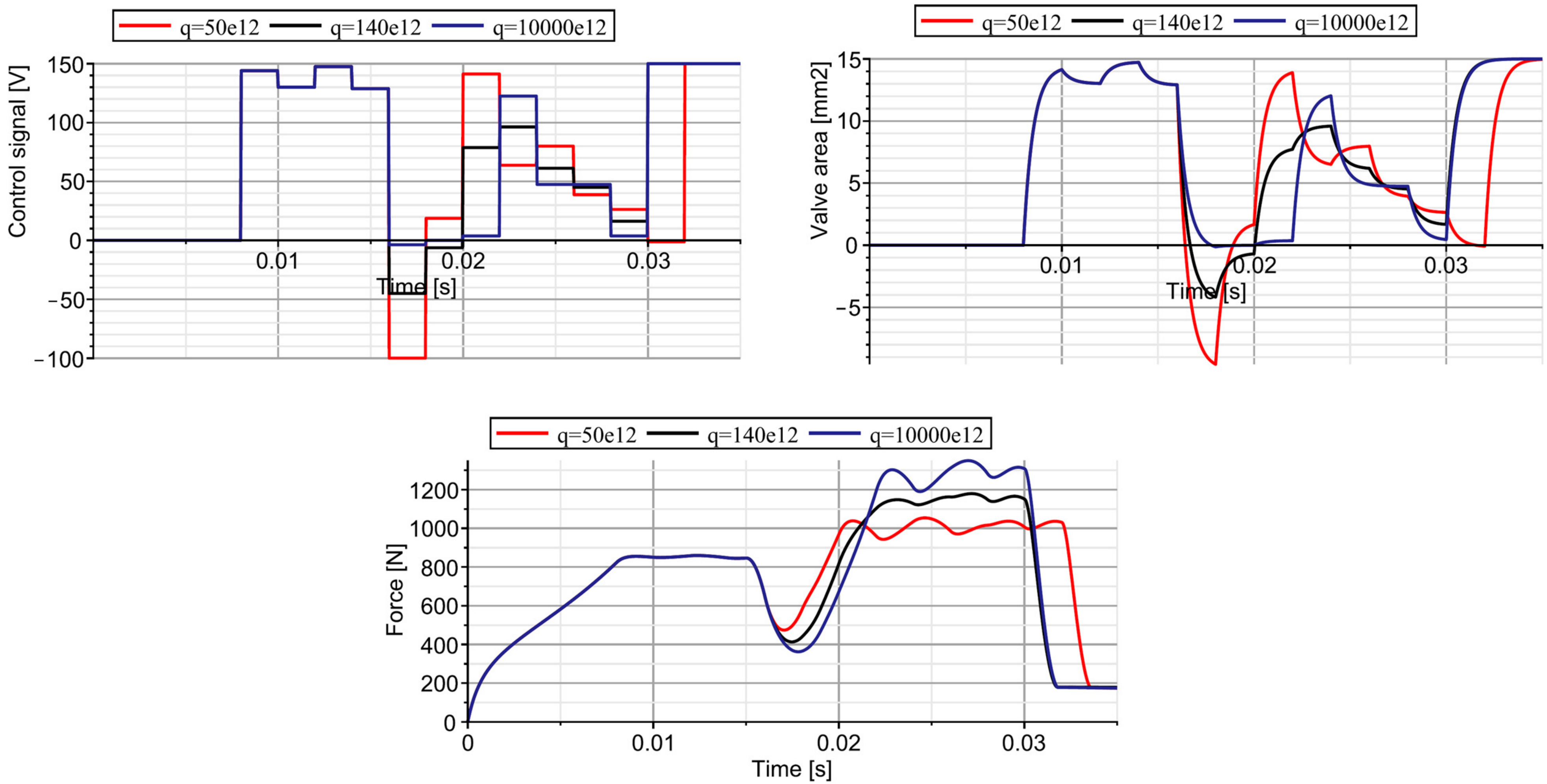




V. Adaptive and predictive control methods

F. Control function parameterisation

- Influence of control cost on operation of active control

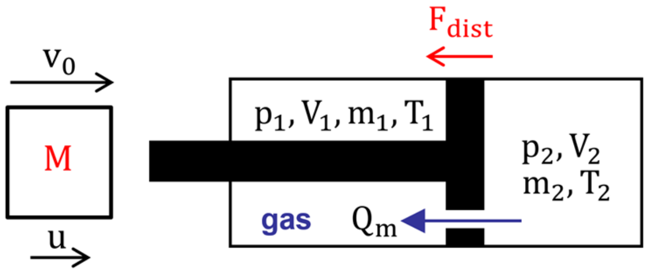


C.Graczykowski, R.Faraj, Extended Identification-based Predictive Control for Optimal Impact Mitigation, submitted to IEEE 61st Conference on Decision and Control (CDC), Cancún, Mexico, December 6-9, 2022

V. Adaptive and predictive control methods

G. System response parameterisation

- Standard problem: absorption of single-impact by semi-active system



Objective: minim. of deceleration by valve area control
Features: unknown mass and disturbance force

- Mathematical formulation: Find $\boldsymbol{\beta}^{opt} = \arg\min \int_{t_i}^{t_i+\Delta t} \left(\ddot{u}(\boldsymbol{\beta}, t) + \frac{\dot{u}(t_i)^2}{2\left(d - u(t_i)\right)} \right)^2 dt$

- Other interpretation: $\boldsymbol{\beta}$ – vector parametrising control function $A_v(\boldsymbol{\beta}, t)$ for which predictive model has analytical solution

- Comparison to control parametrisation: $A_v(\boldsymbol{\beta}, t), \ddot{u}(\boldsymbol{\beta}, t)$ – analytical functions
  set of functions $A_v(\boldsymbol{\beta}, t)$ – very limited

- Determination of function $A_v(\boldsymbol{\beta}, t)$ using $\ddot{u}(\boldsymbol{\beta}, t)$:

Mechanical response: $v(\boldsymbol{\beta}, t) = v(t_i) + \int_{t_i}^{t} \ddot{u}(\boldsymbol{\beta}, t) dt \qquad u(\boldsymbol{\beta}, t) = u(t_i) + \int_{t_i}^{t} v(t_i) dt + \int_{t_i}^{t} \int_{t_i}^{t} \ddot{u}(\boldsymbol{\beta}, t) dt^2$

$$F_p(\boldsymbol{\beta}, t) = -M\ddot{u}(\boldsymbol{\beta}, t) - F_{dist}(t_i, t)$$

G. System response parameterisation

- Determination of function $A_v(\boldsymbol{\beta}, t)$ using $\ddot{u}(\boldsymbol{\beta}, t)$

Thermodynamic response:

$$p_2 A_2 - p_1 A_1 = F_p(\boldsymbol{\beta}, t)$$

$$m_1 + m_2 = m$$

$$\frac{1}{2} M(v(t_i)^2 - v^2) - \int_{u(t_i)}^{u} F_{dist}(t_i, t) du = \Delta U_1 + \Delta U_2 \quad \Rightarrow \quad \begin{matrix} p_i(\boldsymbol{\beta}, t) \\ m_i(\boldsymbol{\beta}, t) \\ T_i(\boldsymbol{\beta}, t) \end{matrix}$$

$$\frac{p_2 V_2^{\kappa}}{m_2^{\kappa}} = \frac{p_2^0 (V_2^0)^{\kappa}}{(m_2^0)^{\kappa}}$$

$$V_1(u) = m_1 R T_1, \;\; p_2 V_2(u) = m_2 R T_2$$

Control function (valve opening):

$$\boxed{\widetilde{A}_v = \frac{dm_1}{dt} [C_v \bar{Q}_m(p_1, p_2, T_2)]^{-1}}$$

$$\widetilde{A}_v = \widetilde{A}_v \left(F_p(t), \frac{dF_p(t)}{dt}, \int_{t_i}^{t} F_p(t) dt, \int_{t_i}^{t} \int_{t_i}^{t} F_p(t) dt^2, M, F_{dist}(t_i, t), p(t_i), u(t_i), v(t_i), t \right)$$

$$\widetilde{A}_v = \widetilde{A}_v \left(\ddot{u}(t), \frac{d\ddot{u}(t)}{dt}, v(t), u(t), M, F_{dist}(t_i, t), p(t_i), t \right)$$

$$\boxed{\widetilde{A}_v = \widetilde{A}_v(\boldsymbol{\beta}, M, F_{dist}, p(t_i), u(t_i), v(t_i), t)}$$

V. Adaptive and predictive control methods

G. System response parameterisation

- Solution of unconstrained optimization problem

$$\beta = \ddot{u}(t_i + \Delta t) \longrightarrow \ddot{u}(\beta, t) = \ddot{u}(t_i) + \frac{\beta - \ddot{u}(t_i)}{\Delta t}(t - t_i)$$

$$\beta^{opt} = \arg\min \int_{t_i}^{t_i+\Delta t} \left(\ddot{u}(t_i) + \frac{\beta - \ddot{u}(t_i)}{\Delta t}(t - t_i) + \frac{\dot{u}(t_i)^2}{2(d - u(t_i))} \right)^2 dt$$

$$\boxed{\beta^{opt} = \ddot{u}^{opt}(t_i + \Delta t) = -\frac{\ddot{u}(t_i)}{2} + \frac{3}{4}\frac{\dot{u}(t_i)^2}{(d - u(t_i))}}$$

Distance from the optimal path decreases twice during each control step

- Solution of constrained optimization problem

Standard constraints: $A_v^{min} \leq \tilde{A}_v(\boldsymbol{\beta}, t) \leq A_v^{max}$, $V_{A_v}^{min} \leq \frac{d\tilde{A}_v(\boldsymbol{\beta},t)}{dt} \leq V_{A_v}^{max}$

A. Constraints reached at the ends of control step:

$$A_v^{min} \leq \tilde{A}_v(\boldsymbol{\beta}, t_i) \leq A_v^{max}, \quad A_v^{min} \leq \tilde{A}_v(\boldsymbol{\beta}, t_i + \Delta t) \leq A_v^{max}$$

B. Constraints reached in the middle of control step:

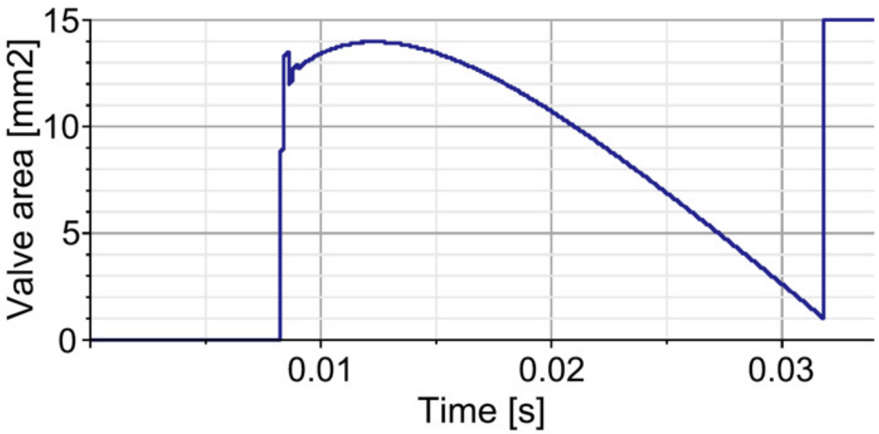
$$t_{ext}(\boldsymbol{\beta}) = t(\boldsymbol{\beta}) \mid \frac{d\tilde{A}_v(\boldsymbol{\beta},t)}{dt} = 0, \quad A_v^{min} \leq \tilde{A}_v\big(\boldsymbol{\beta}, t_{ext}(\boldsymbol{\beta})\big) \leq A_v^{max}$$

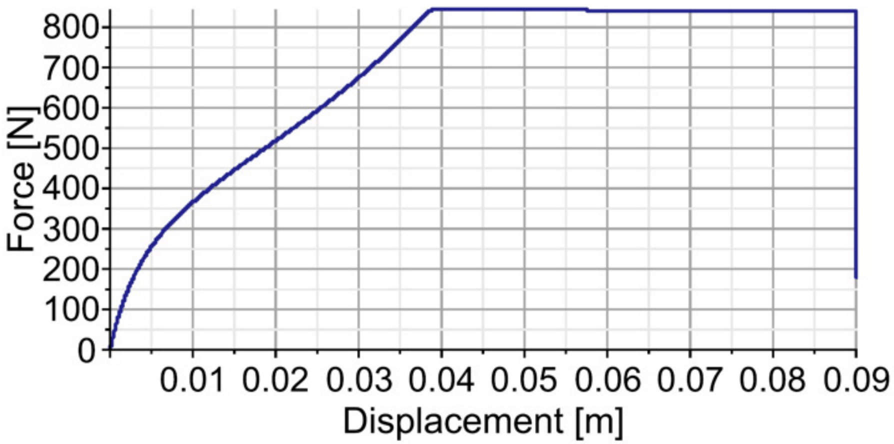
Constrained problem solved using Lagrange multipliers and KKT conditions

V. Adaptive and predictive control methods

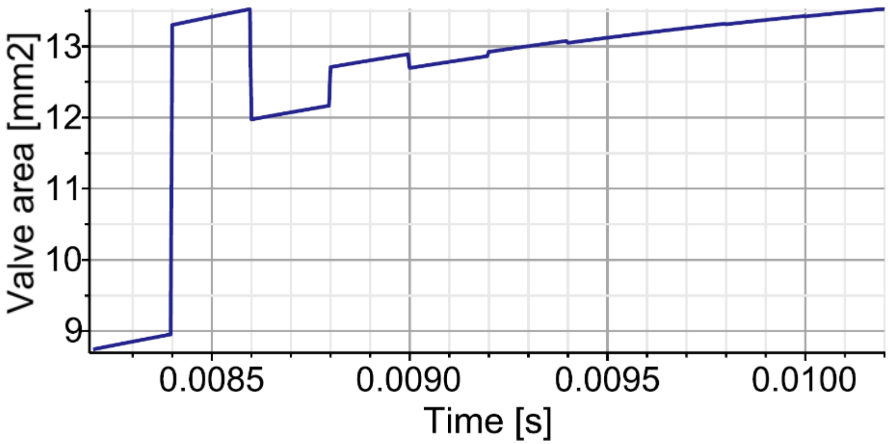
G. System response parameterisation

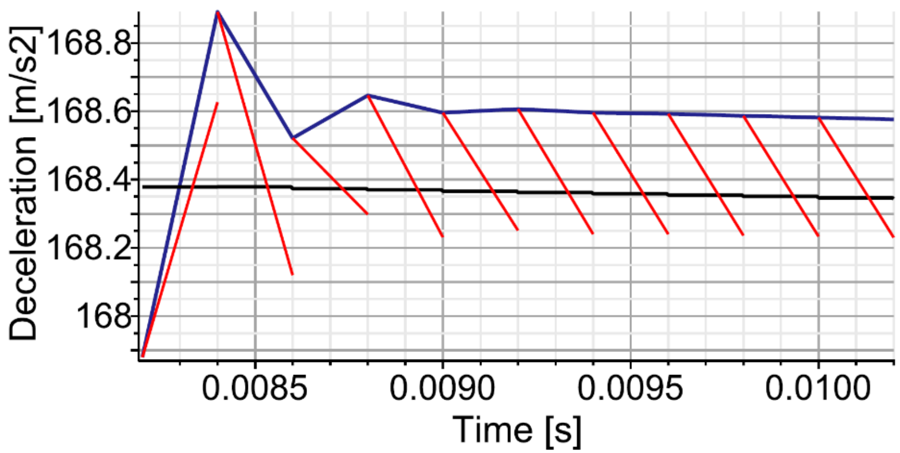
Operation during entire process





Details of method operation

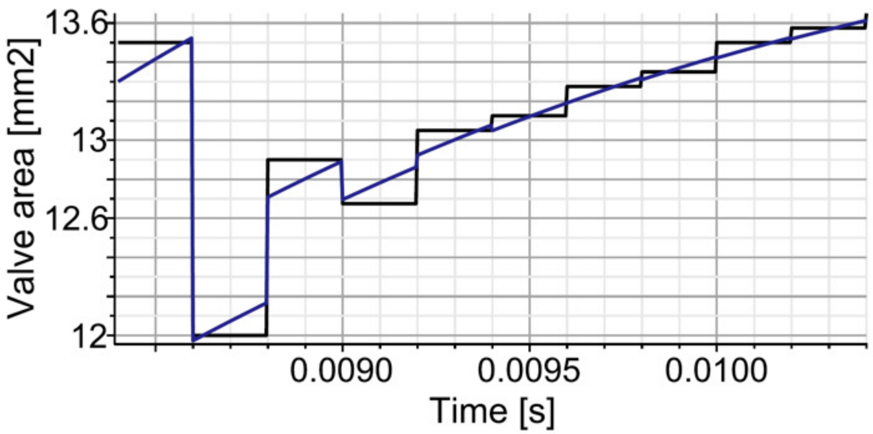


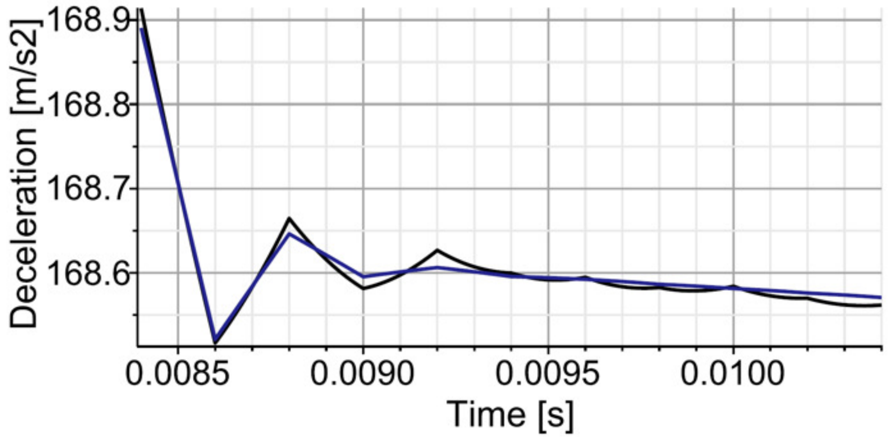


V. Adaptive and predictive control methods

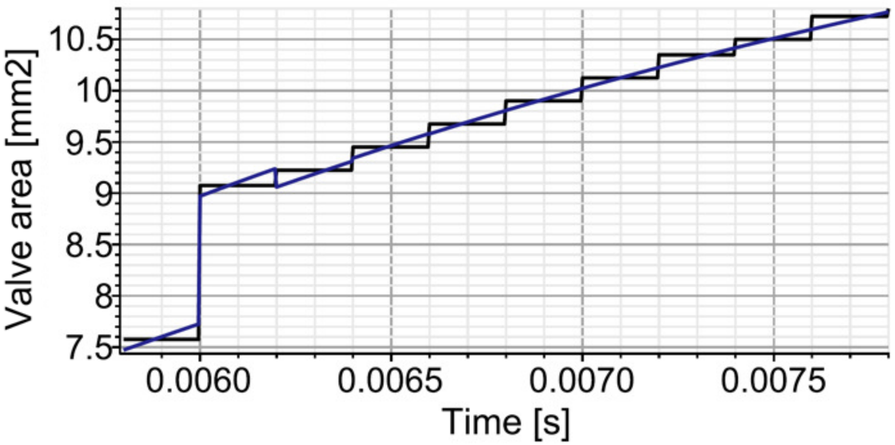
G. System response parameterisation

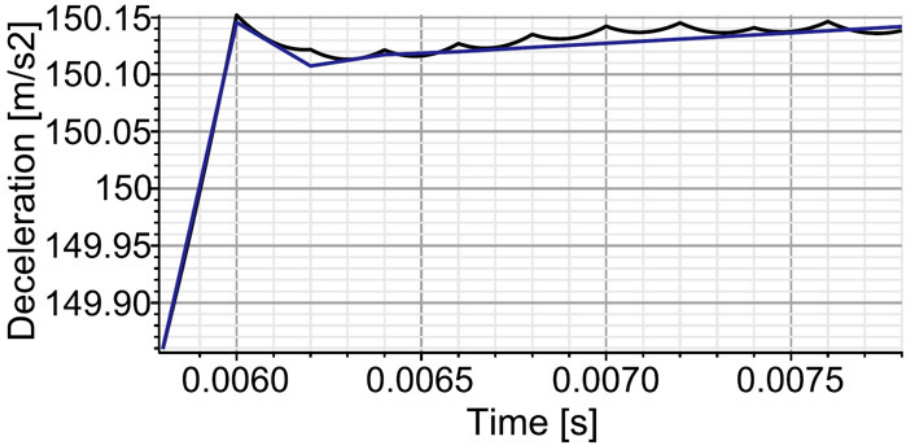
Comparison of methods with control and response parameterisation – case of elastic disturbance





Comparison of methods with control and response parameterisation – case of viscous disturbance

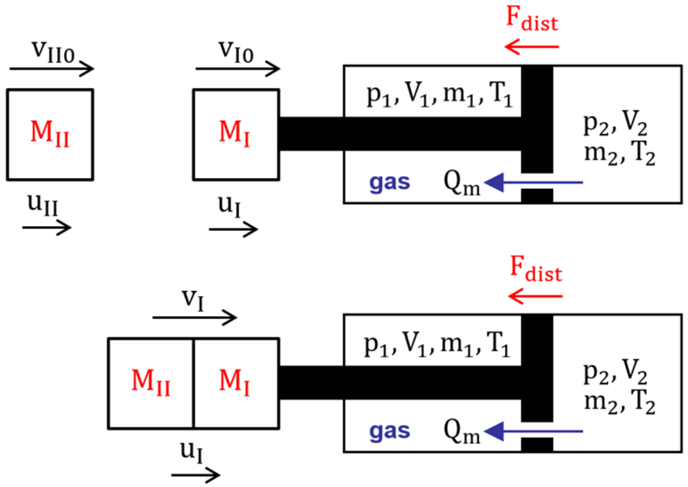




V. Adaptive and predictive control methods

G. System response parameterisation

- Advanced problem: absorption of double-impact by semi-active system



Objective: minim. of deceleration by valve area control

Features: unknown mass M_I and M_{II},
unknown disturbance force F_{dist}

- Mathematical formulation: Find $\boldsymbol{\beta}^{\mathbf{opt}} = \arg\min \int_{t_i}^{t_i+\Delta t} \left(\ddot{u}_I(\boldsymbol{\beta}, t) + \frac{\dot{u}_I(t_i)^2}{2\left(d - u_I(t_i)\right)} \right)^2 dt$

- Applied models:
  - system model: 2 DOF (mass M_I and M_{II}) + contact force
  - predictive model: 1 DOF (unknown mass M_I) + external force F_{ext}

- System identification: $M_I \ddot{u}(t) + F_p(t) + F_{dist}(t) = F_{ext}(t)$

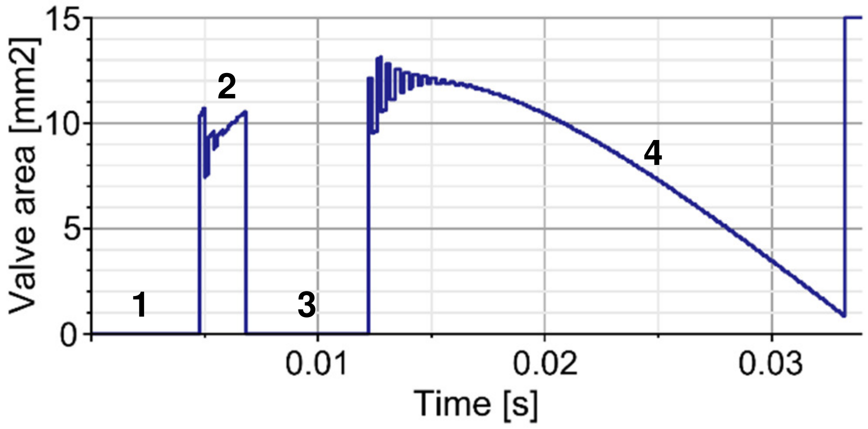
$$\underbrace{\left(M_I - \frac{F_{ext}(t) - F_{dist}(t)}{\ddot{u}_I(t)} \right)}_{M_{eq}} \ddot{u}_I(t) + F_p(t) = 0 \quad \rightarrow \quad M_{eq} = -\frac{F_p(t_i)}{\ddot{u}_I(t_i)}$$

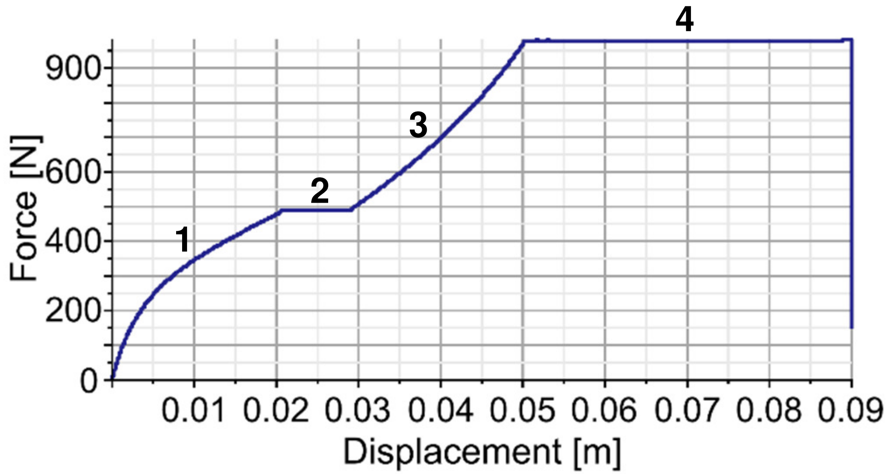
- Solution of the optimization problem: identical as in previous example

V. Adaptive and predictive control methods

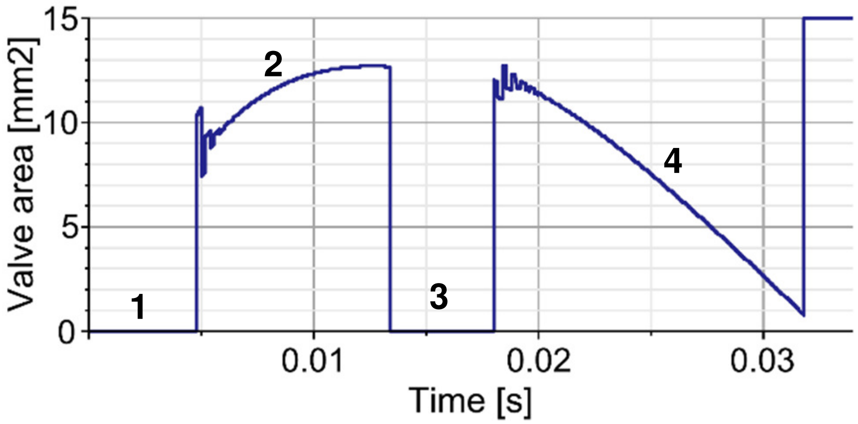
G. System response parameterisation

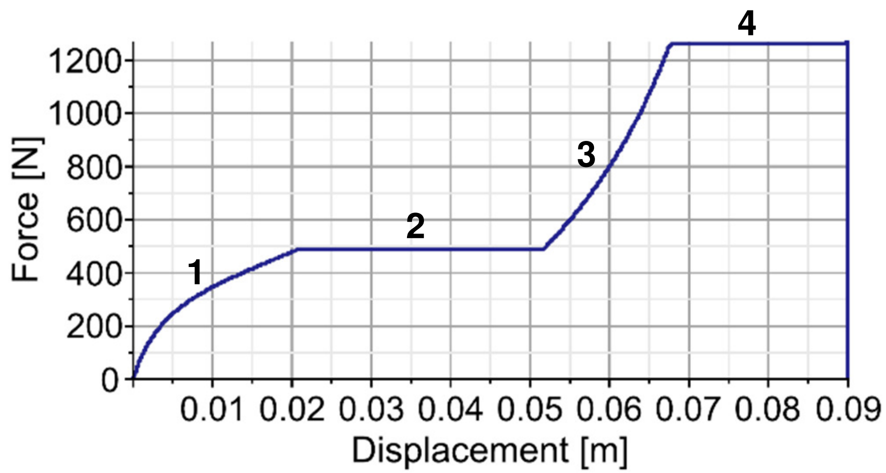
Double-impact with inelastic collision, no disturbance: Scenario 1





Double-impact with inelastic collision, no disturbance: Scenario 2

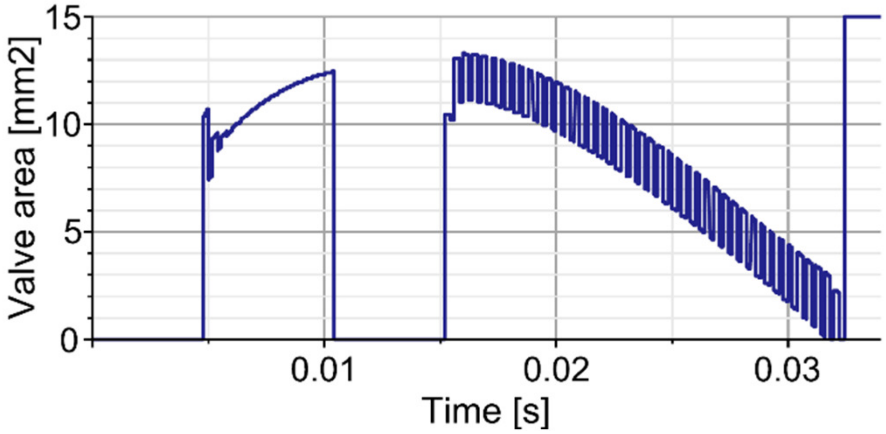


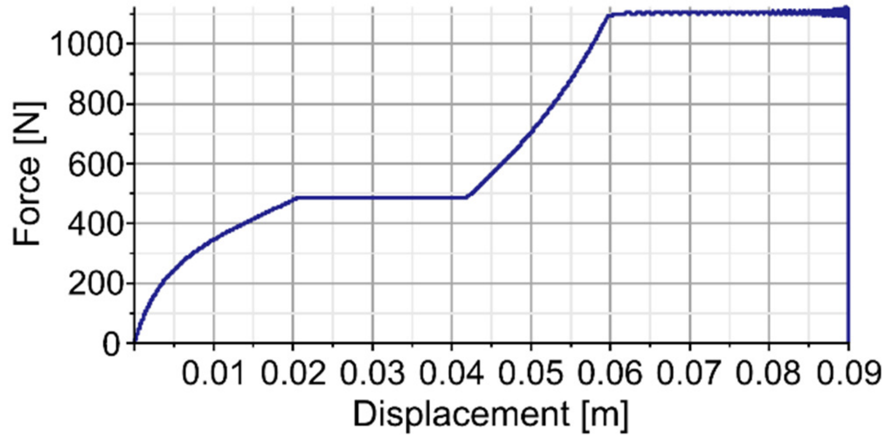


V. Adaptive and predictive control methods

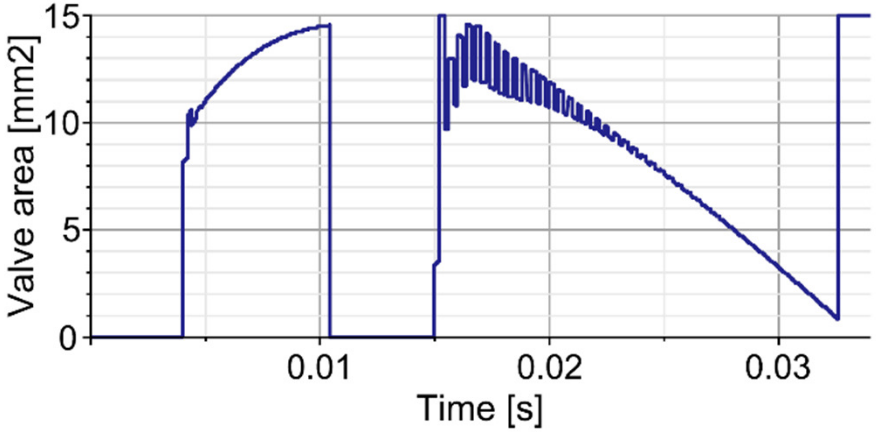
G. System response parameterisation

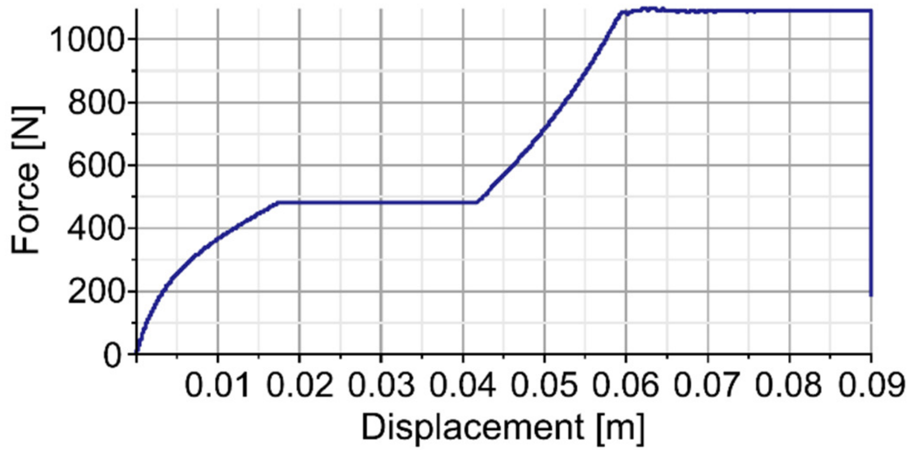
Double-impact with partially inelastic collision, no disturbance:





Double-impact with partially inelastic collision, elastic disturbance force





C.Graczykowski, R.Faraj, Adaptive Model Predictive Control of semi-active shock-absorbers under impact excitation, submitted to Structural Control and Health Monitoring, 2022

VI. Summary and conclusions

Summary:

1. The problem of **Adaptive Impact Absorption** with the use of **controllable dampers** was considered.
2. The **state-dependent formulation of AIA problem** was proposed.
3. The formulation was investigated using Pontryagin's principle and direct methods.
4. The **adaptive control algorithms** based on concept of MPC, system identification, and various methods of control determination were proposed and tested.

Conclusions:

1. State-dependent formulation and MPC approach are well adjusted to AIA problems with unknown system parameters, excitations and disturbances.
2. Algorithms with arbitrary change of control in time provide optimal system response, but require intensive control actions.
3. Algorithms with control parametrisation enable significant decrease of the control cost.
4. Algorithms with response parametrisation additionally allow to reduce the numerical cost.

Dziękuję za uwagę!