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Biomimetic Structural Optimization Method

New paradigm for shape and topology optimization

Sekcja Optymalizacji KM PAN

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Outline

Biomimetic Structural Optimization Method
New paradigm for shape and topology optimization

- The Structural Optimization Problem
- The standard approach to compliance minimization
- The biomimetic approach to compliance minimization
- The comparison of the standard and modified approach
- The new paradigm for compliance minimization – different topologies for different load magnitudes
- The new paradigm for compliance minimization – different topologies for different materials
- Conclusions

The Structural Stiffness Maximization

The goal is to maximize the stiffness of a structure, that is minimization of the functional

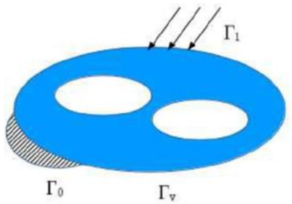
$$J(\Omega) = \int_{\Gamma_1} t.u\,ds$$

under constraints

$$\int_{\Omega} dx - V_0 = 0$$

and state equations

$$\text{div}\sigma(u) = 0 \quad \text{in} \quad \Omega$$



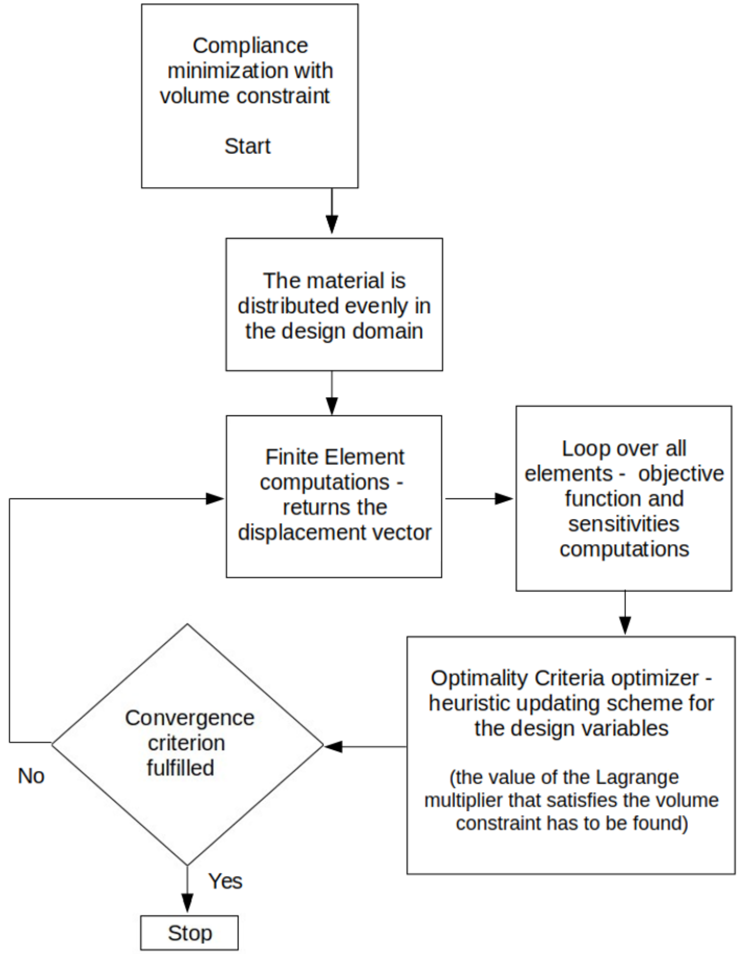
$$\sigma(u).n = t \quad \text{on} \quad \Gamma_1$$

$$\sigma(u).n = 0 \quad \text{on} \quad \Gamma_v$$

$$u = 0 \quad \text{on} \quad \Gamma_0$$

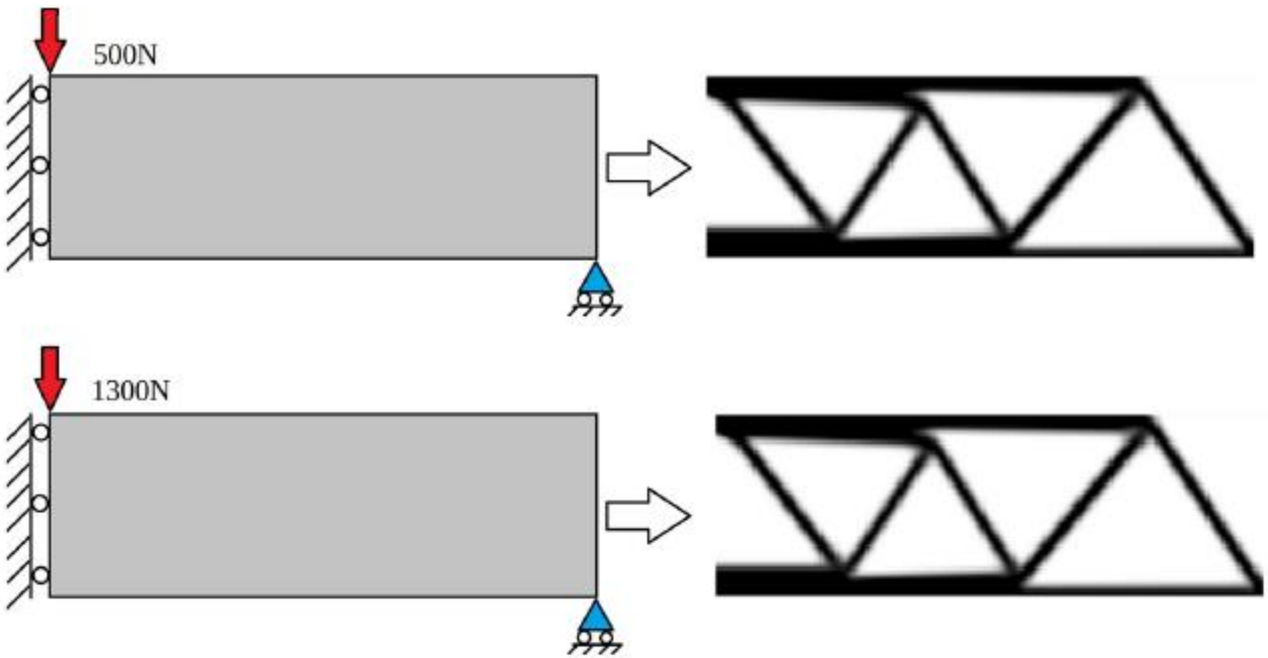
Here, Ω represents the domain of the elasticity system, u the displacement, V_0 a given volume, Γ_0 part of the boundary with Dirichlet condition, Γ_1 part of the boundary loaded by traction forces, Γ_v part of the boundary subject to modification.

The standard approach to compliance minimization



IMP Solid Isotropic Material with Penalization

The standard approach to compliance minimization



The MATLAB based topology optimization code results for the MBB-beam example – different values of the loading force, similar results.

The Structural Stiffness Maximization

The biomimetic approach to compliance minimization

To justify an assumption about constant energy density on the structural surface the Lagrange function for the problem under considerations is defined in the form

$$L(\Omega_t, \lambda) = \int_{\Gamma_1} \mathbf{t}.\mathbf{u}_t \, ds + \lambda \left[\int_{\Omega_t} dx - V_0 \right]$$

rewrited state equation in the weak form

$$-\int_{\Omega_t} \boldsymbol{\sigma}(\mathbf{u}_t) : \boldsymbol{\varepsilon}(\boldsymbol{\varphi}) \, dx + \int_{\Gamma_1} \mathbf{t}.\boldsymbol{\varphi} \, ds = 0$$

then the shape derivative of both Lagrange function and weak state equation using speed method [Sokołowski J., Zolesio J., Introduction to Shape Optimization. Shape Sensitivity Analysis, Springer-Verlag, 1992] is taken and for fixed vector field V(x) at the local minimum this first derivative should vanish, then

$$[L(\Omega, \lambda)]' = \int_{\Gamma_1} \mathbf{t}.\mathbf{u}' \, ds + \lambda \int_{\Gamma_v} \mathbf{V}.\mathbf{n} \, ds = 0 \qquad \int_{\Gamma_1} \mathbf{t}.\mathbf{u}' \, ds = -\int_{\Gamma_v} \boldsymbol{\sigma}(\mathbf{u}) : \boldsymbol{\varepsilon}(\mathbf{u}) \mathbf{V}.\mathbf{n} \, ds$$

for the stationary point this should hold for any vector field V(x) on Γ_V, then

The Structural Stiffness Maximization

$$\int_{\Gamma_v} [\lambda - \boldsymbol{\sigma}(\mathbf{u}) : \boldsymbol{\varepsilon}(\mathbf{u})] \mathbf{V}.\mathbf{n}\, ds = 0$$

$$\boldsymbol{\sigma}(\mathbf{u}) : \boldsymbol{\varepsilon}(\mathbf{u}) = \lambda = \text{const.}$$

Nowak M., Sokołowski J. & Żochowski A., Justification of a certain algorithm for shape optimization in 3D elasticity, Struct Multidiscipl Optim, 57, pp. 721–734, https://doi.org/10.1007/s00158-017-1780-7, 2018

To maximize the structural stiffness, the strain energy density on the structural surface should be constant.

The value of λ is not known. The assumed value of the strain energy density on the part of the boundary subject to modification could be related to the material properties.

Change in the assumed value of the strain energy density results in change of the structural form – topology and volume.

In this way, the final structural volume results from the optimization procedure.

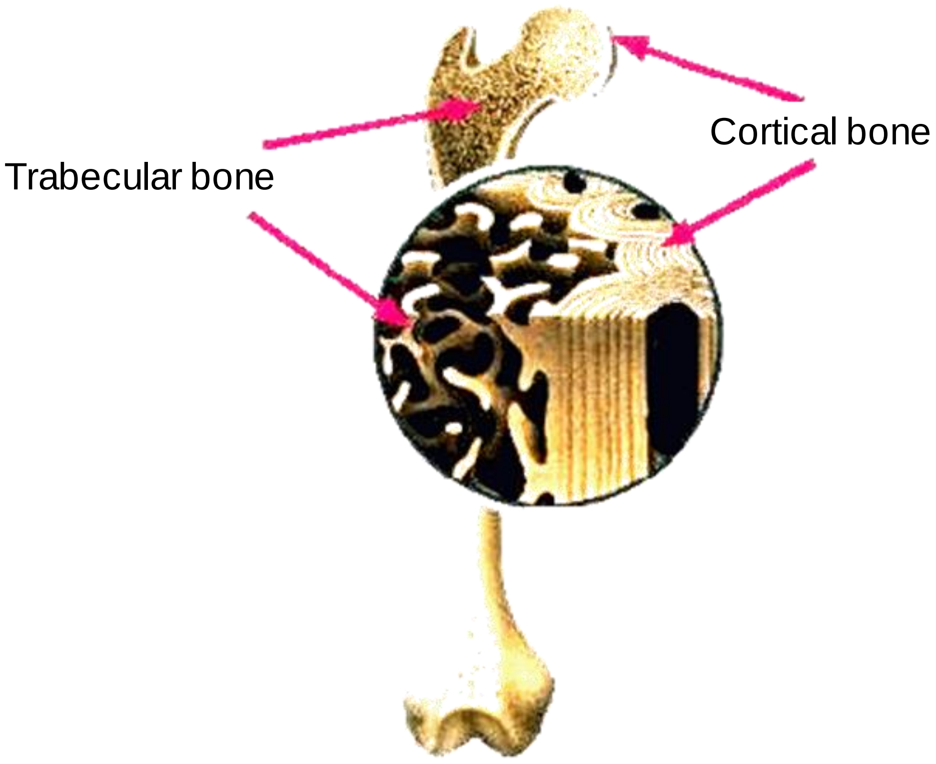
Instead of imposing volume constraint it is possible to parameterize shapes by the assumed energy density on the structural surface, which may be quite accurately predicted from the yield criteria.

But how to realize it in practice? → Biomimetics …

Trabecular bone surface adaptation

- Bone remodeling – tissue growth and resorbtion balance on the surface

- Mechanical stimulation – tissue mechanosensation



The Biomechanical Regulatory Model

'Mechanosensitivity'

On the surface only!

Huiskes Ruimerman

'Regulatory model'

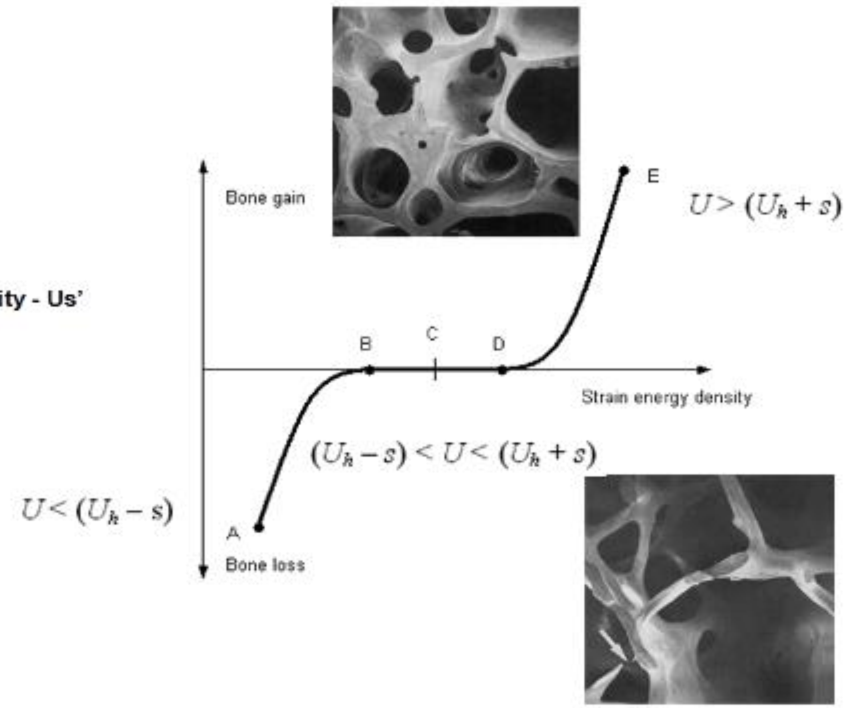
(Huiskes et al. 2003)

'Homeostatic strain energy density - Us'

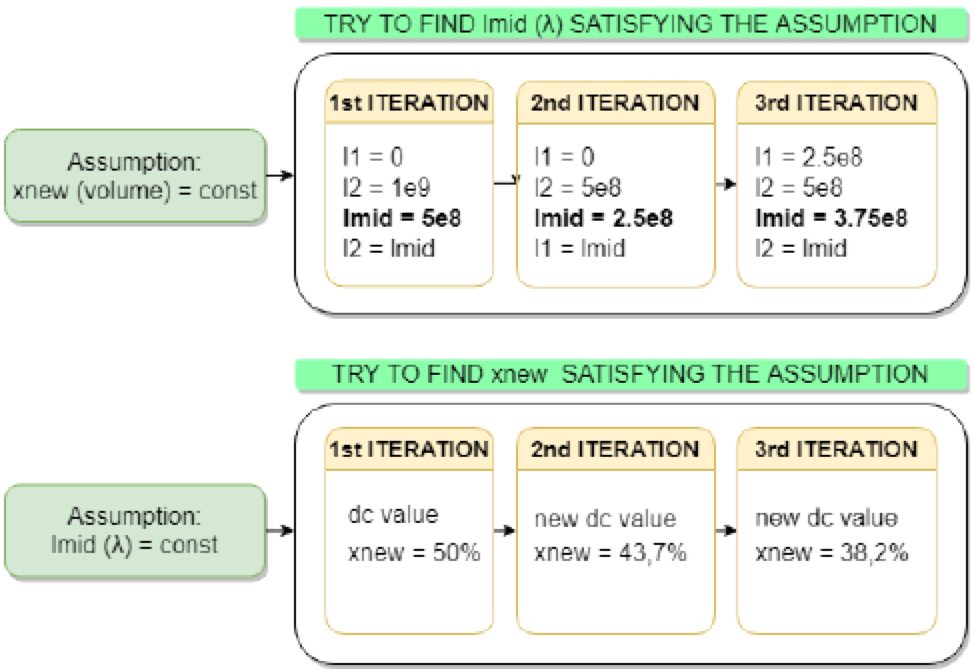
Perfect balance between resorbtion and new tissue creation.

'Lazy zone'

Carter (Carter et al. 1989)

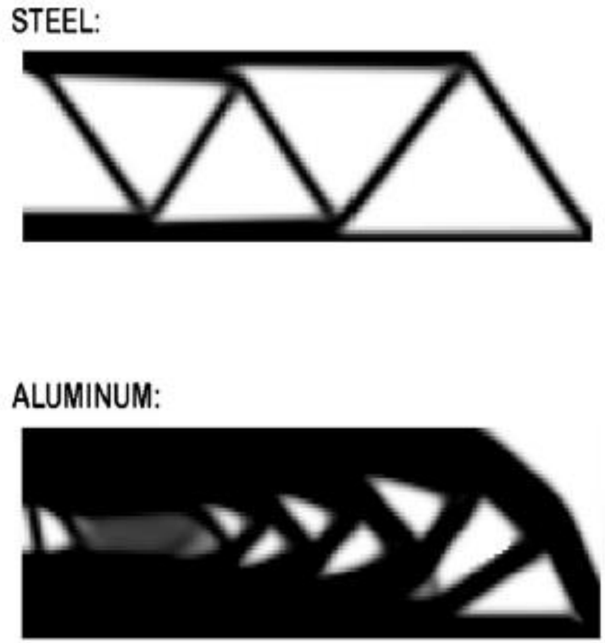


The modified approach



On the top row - the original optimization procedure (***xnew, denoting the volume constrain –*** constant and ***lmid, denoting the Lagrange multiplier – different value for each iteration).***
On the bottom row modified optimization procedure (***xnew – different value for each iteration and variable different value for*** each iteration and ***lmid - constant) optimization procedures***

The biomimetic approach



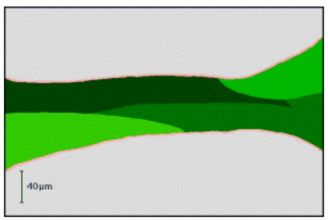
- steel: Young modulus E=210GPa, Poisson's ratio ν=0.30, the value of the Lagrange multiplier lmid=620MPa,
- aluminum (as for alloy 1050A1): Young modulus E=69GPa, Poisson's ratio ν=0.33, and the value of the Lagrange multiplier was assumed for this material lmid=90MPa.

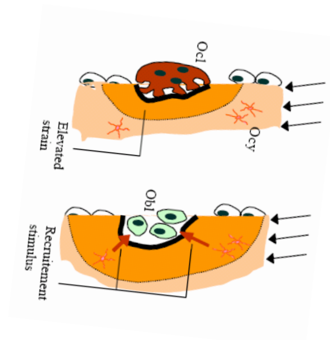
The new paradigm for compliance minimization – different topologies for different materials

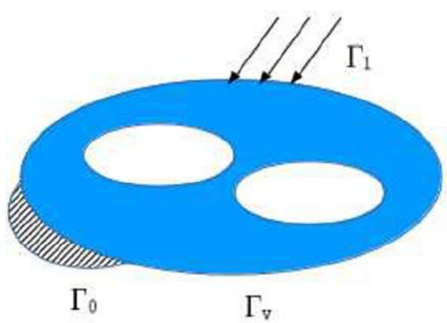
The Structural Stiffness Maximization

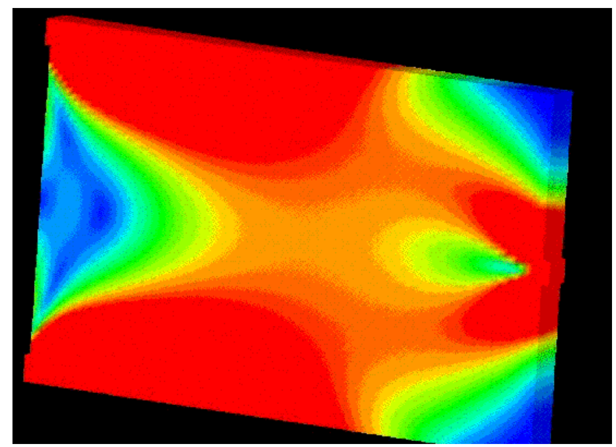
$$\sigma(u) : \varepsilon(u) = \lambda = \text{const}$$

Nowak M., Sokołowski J. & Żochowski A., Justification of a certain algorithm for shape optimization in 3D elasticity, Struct Multidiscipl Optim, 57, pp. 721–734, https://doi.org/10.1007/s00158-017-1780-7, 2018









The Biomimetic Approach to Shape Modification

The function F(z) is penalizing the deviation of z from λ is and taking into account the 'lazy zone' - insensitivity zone the function is defined

$$F(z) = \begin{cases} z < -s & : (z+s)^2 \\ -s \le z \le s & : 0 \\ z > s & : (z-s)^2 \end{cases}$$

Using this function it is possible to describe the biomimetic strategy of equalizing the strain energy density on the structural surface by equivalent minimization of the funcional

$$J_\lambda(\Omega) = \int_{\Gamma_V} F\big(\sigma(u) : \varepsilon(u) - \lambda\big) ds$$

After taking the shape derivative, the derivative has the form

$$J_\lambda(\Omega)' = \int_{\Gamma_V} [\sigma(u) : \varepsilon(p) + \frac{\partial F}{\partial n} + \kappa F] V.n ds$$

The Biomimetic Approach to Shape Modification

$$J_{\lambda}(\Omega)' = \int_{\Gamma_v} [\sigma(u) : \varepsilon(p) + \frac{\partial F}{\partial n} + \kappa F] V.n ds$$

When the expression in square brackets is negative, then the material should be added (V.n > 0), otherwise the material should be removed from the structural surface(V.n < 0)

The first term in brackets represents non-local influence of boundary modification, the second term describes the change of integral due to spatial variability of F, and the third takes into account the increase or decrease of the area of the surface itself.

The biomimetic, heuristic algorithm for the structural optimization with shapes parameterization by the assumed energy density on the structural surface can be described as follows :

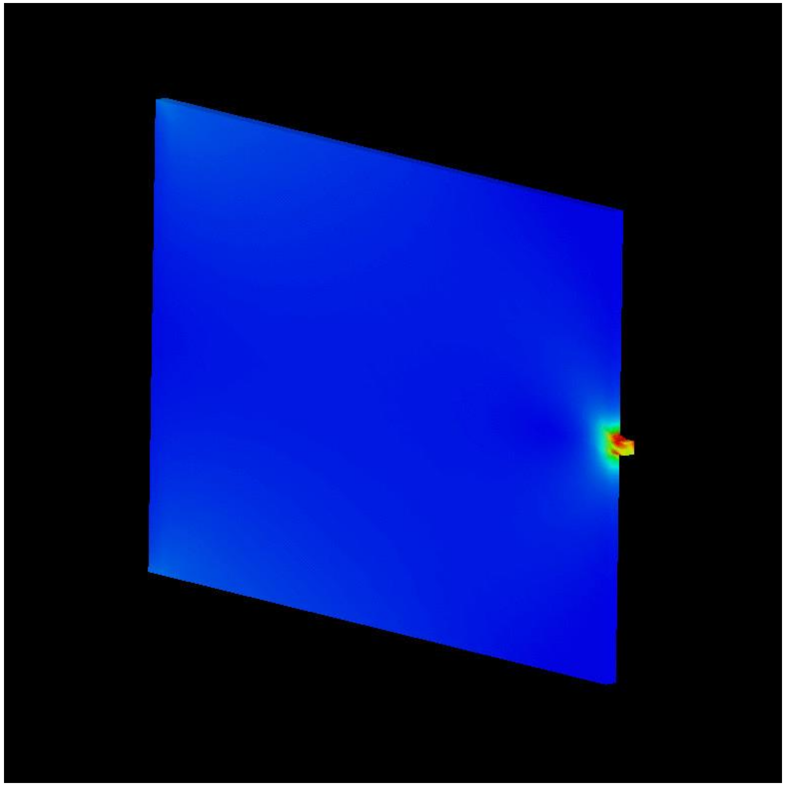
– it is assumed that the energy density has a constant λ value on *Γv*,
– if at a given point on *Γv* this density is bigger than λ + s then the boundary in moved outside,
– if at a given point on *Γv* this density is smaller than λ − s then the boundary in moved inside,
– these steps are repeated until equilibrium is achieved,
– the value of λ is modified if the final design is unsatisfactory.

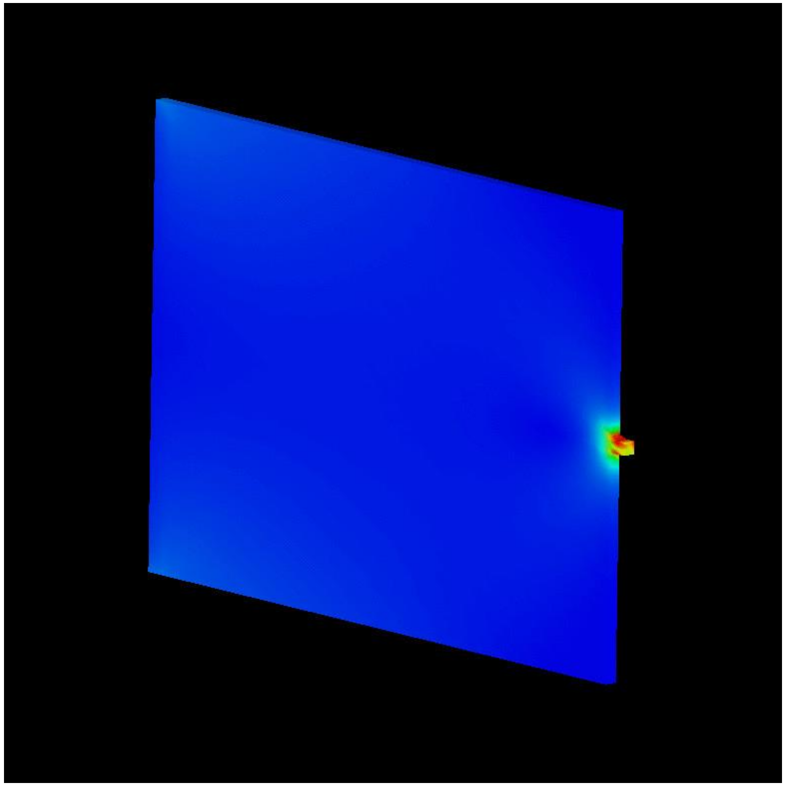
The improvement – the surface curvature.

– if κ > 0 and given point is outside the lazy zone, then after biomimetic modification the boundary is additionally moved inside by 50% of the biomimetic step,
– if κ < 0 and given point is outside the lazy zone, then after biomimetic modification the boundary is additionally moved outside by 50% of the biomimetic step.

$$\sigma(u) : \varepsilon(u) = \lambda = const$$

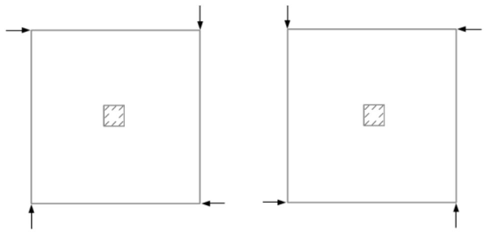
The Biomimetic Approach to Shape Modification

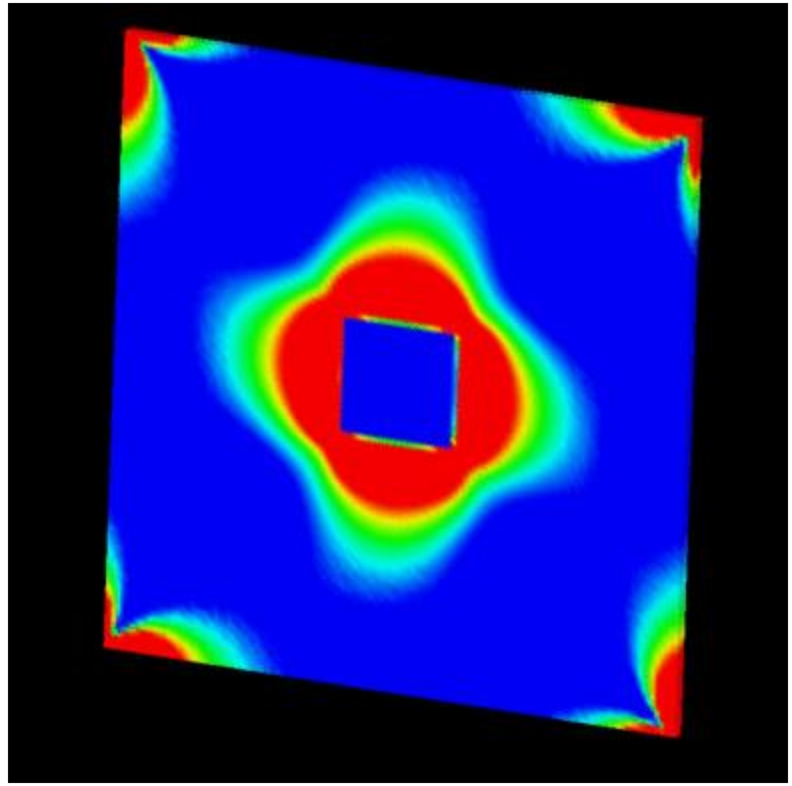




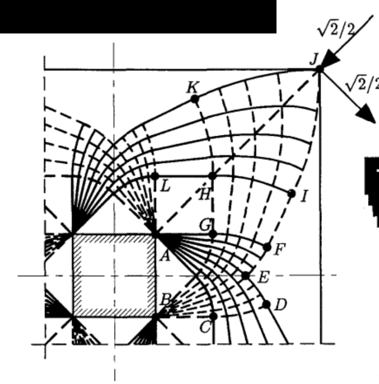
Bending of the cantilever beam .
Left: optimization results without the curvature measuring term. Right: optimization results with the curvature measuring term.

Optimization with Multiple Load Conditions - Benchmark Problems





Rozvany, G, Exact analytical solutions for some popular benchmark problems in topology optimization, Structural optimization, vol. 15, pp. 42–48, (1998)



Beckers M., Topology optimization using a dual method with discrete variables, Structural optimization, pp. 14–24, (1999)

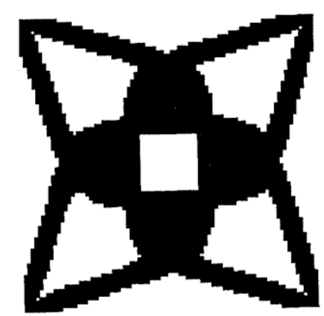
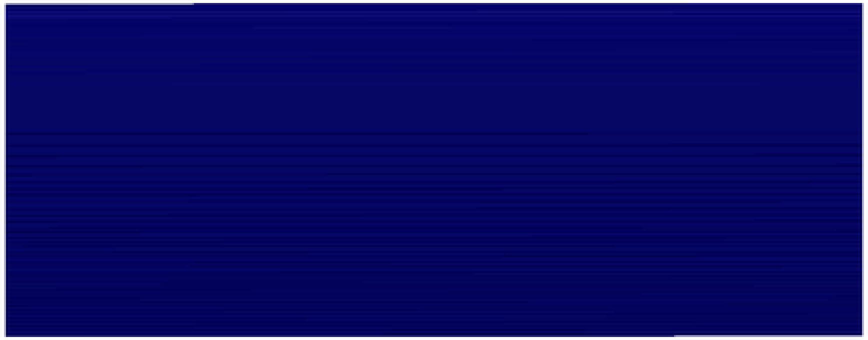
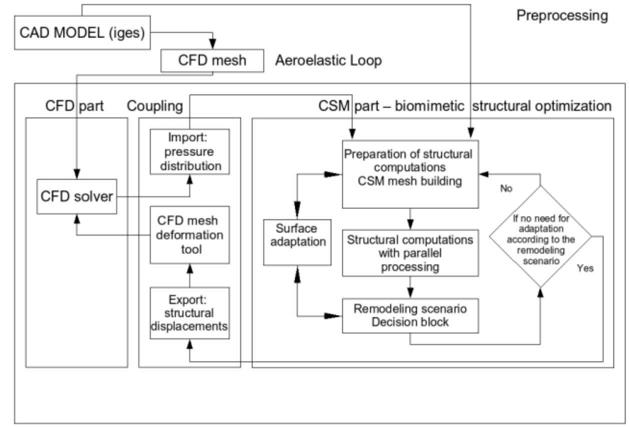
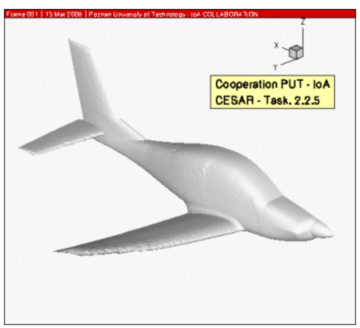


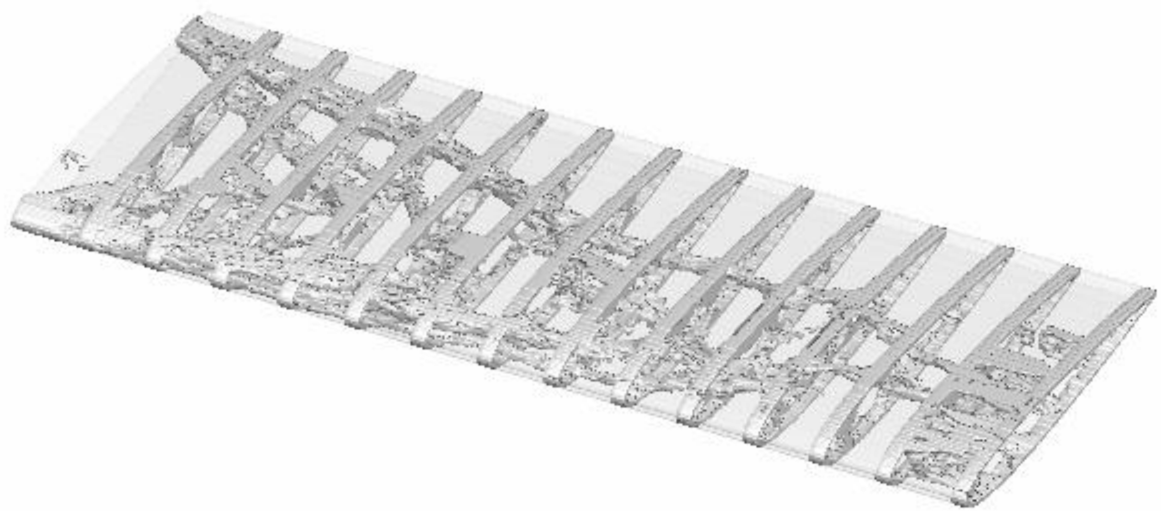
Fig. 16. Solution - multiple load cases

Practical Example - Coupling the Optimization Procedure with Aeroelasticity









Conclusions

Unique features of the presented method, which provide new possibilities in the area of structural optimization, like:

- the domain independence,
- functional configurations during the process of optimization,
- possibility to solve the multiple load problems,

allow to comprise optimizations of shape, and topology with no need to define parameters.

The presented method is able to produce results similar to the standard method of topology optimization and can be useful in mechanical design, especially when functional structures are needed during the optimization process.

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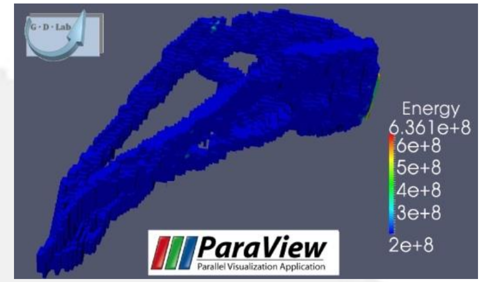

DECYZJA Nr DWP/TECHMATSTRATEG-III/136/2020
Dyrektora Narodowego Centrum Badań i Rozwoju
w sprawie przyznania środków finansowych

Bioniczne, lekkie węzły strukturalne wytwarzane przyrostowo dla przemysłu motoryzacyjnego

w ramach III konkursu Strategicznego Programu Badań Naukowych i Prac Rozwojowych „Nowoczesne technologie materiałowe" TECHMATSTRATEG

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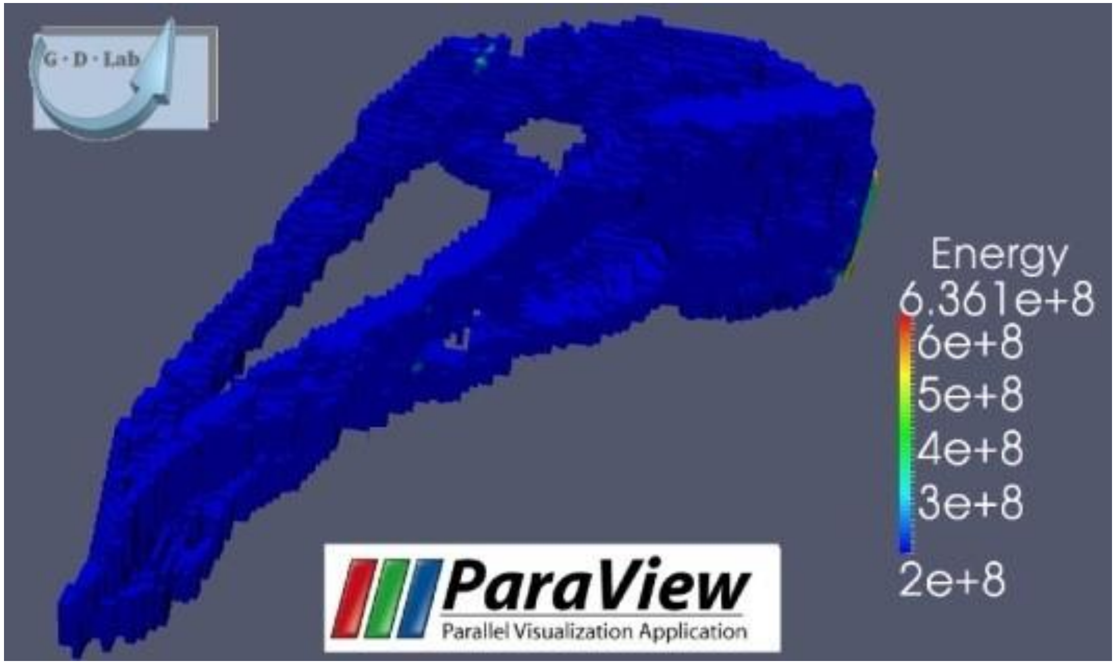
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